

Unit :2 Linear Applications of Op-Amp

• **Inverting amplifier:**

- The signal to be amplified (V_s) has been connected to the inverting terminal via the resistance R_1 . Change in output. The other resistor R_f connected between the output and inverting terminals is called as feedback resistance. It introduces a negative feedback.
- Closed loop gain $A_{vf} = V_0/V_s$
- $= -R_f/R_1$
- The negative sign indicates that there is a phase shift of 180 degree between input and output voltage.

• **Non inverting amplifier:**

- The negative feedback is incorporated in this circuit via a feedback resistor R_f which is connected between output and inverting terminal of OP-AMP.
- Closed loop voltage gain A_{vf} is given as:
- $A_{vf} = V_0/V_s = 1 + R_f/R_1$.

Comparison of the ideal inverting and non-inverting op-amp

Ideal Inverting amplifier	Ideal Non-inverting amplifier
1. Voltage gain = $- R_f/R_1$	1. Voltage gain = $(1+R_f/R_1)$
2. The output is inverted with respect to input	2. No phase shift between input and output
3. The voltage gain can be adjusted as greater than, equal to or less than one	3. The voltage gain is always greater than one
4. The input impedance is R_1	4. The input impedance is very large

Example 2.1

Design an amplifier with a gain of -10 and input resistance equal to $10\text{ k}\Omega$.

Solution

Since the gain of the amplifier is negative, an inverting amplifier has to be made.

In Fig. 2.5 (a) choose $R_1 = 10\text{ k}\Omega$

$$\begin{aligned}\text{Then } R_f &= -A_{CL} R_1 \text{ (from Eq. 2.4)} \\ &= -(-10) \times 10\text{ k}\Omega = 100\text{ k}\Omega\end{aligned}$$

Example 2.2

In Fig. 2.5 (b), $R_1 = 10\text{ k}\Omega$, $R_f = 100\text{ k}\Omega$, $v_i = 1\text{ V}$. A load of $25\text{ k}\Omega$ is connected to the output terminal. Calculate (i) i_1 (ii) v_o (iii) i_L and (iv) total current i_o into the output pin.

Solution

$$\text{(i) } i_1 = \frac{v_i}{R_1} = \frac{1\text{ V}}{10\text{ k}\Omega} = 0.1\text{ mA}$$

$$\text{(ii) } v_o = -\frac{R_f}{R_1} v_i = -\frac{100\text{ k}\Omega}{10\text{ k}\Omega} 1\text{ V} = -10\text{ V}$$

$$\text{(iii) } i_L = \frac{v_o}{R_L} = \frac{10\text{ V}}{25\text{ k}\Omega} = 0.4\text{ mA}$$

The direction of i_L is shown in Fig. 2.5 (b).

- (iv) i_1 as calculated above is 0.1 mA .
Therefore, total current $i_o = i_1 + i_L = 0.1\text{ mA} + 0.4\text{ mA} = 0.5\text{ mA}$. In an inverting amplifier, for a +ive input, output will be -ive, therefore the direction of i_o is as shown in Fig. 2.5 (b).

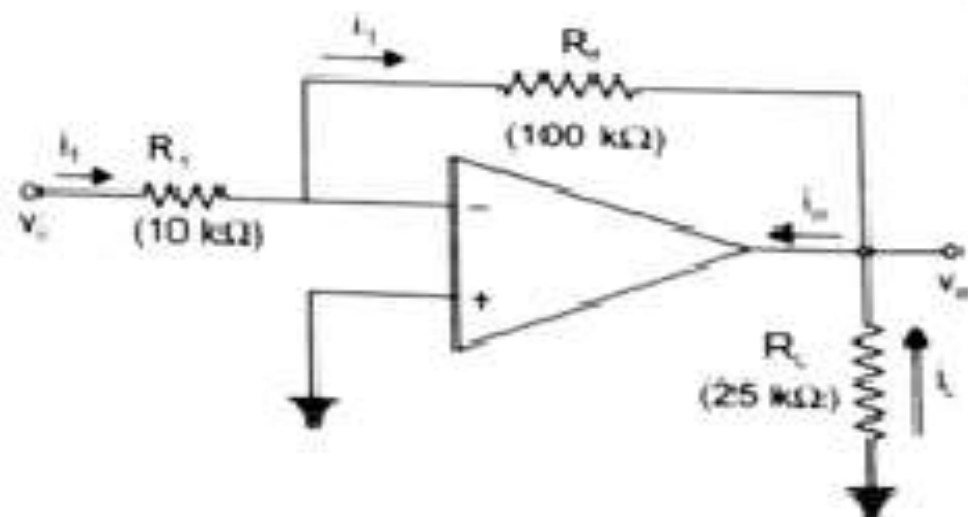


Fig. 2.5 (b) Circuit for Example 2.2

Example 2.3

Design an amplifier with a gain of +5 using one op-amp.

Solution

Since the gain is positive, we have to make a non-inverting amplifier. In Fig. 2.7 (a) select $R_1 = 10 \text{ k}\Omega$. Then from Eq. (2.20)

$$A_{CL} = 1 + R_f/R_1$$

or,
$$5 = 1 + R_f/10 \text{ k}\Omega$$

or,
$$R_f = 4 \times 10 \text{ k}\Omega = 40 \text{ k}\Omega$$

Example 2.4

In the circuit of Fig. 2.7 (a), let $R_1 = 5 \text{ k}\Omega$, $R_f = 20 \text{ k}\Omega$ and $v_i = 1 \text{ V}$. A load resistor of $5 \text{ k}\Omega$ is connected at the output as in Fig. 2.5 (b). Calculate, (i) v_o , (ii) A_{CL} , (iii) the load current i_L , (iv) the output current i_o indicating proper direction of flow.

Solution

$$(i) \quad v_o = \left(1 + \frac{R_f}{R_1}\right) v_i = \left(1 + \frac{20 \text{ k}\Omega}{5 \text{ k}\Omega}\right) (1 \text{ V}) = 5 \text{ V}$$

$$(ii) \quad A_{CL} = \frac{v_o}{v_i} = \frac{5 \text{ V}}{1 \text{ V}} = 5$$

$$(iii) \quad i_L = \frac{v_o}{R_L} = \frac{5 \text{ V}}{5 \text{ k}\Omega} = 1 \text{ mA}$$

$$(iv) \quad i_1 = \frac{v_i}{R_1} = \frac{v_o - v_i}{R_f} = 0.2 \text{ mA}$$

Therefore, $i_o = i_L + i_1 = 1 \text{ mA} + 0.2 \text{ mA} = 1.2 \text{ mA}$

The op-amp output current i_o flows outwards from the output junction.

Summing, Scaling, and Averaging Amplifiers (Three Inputs)

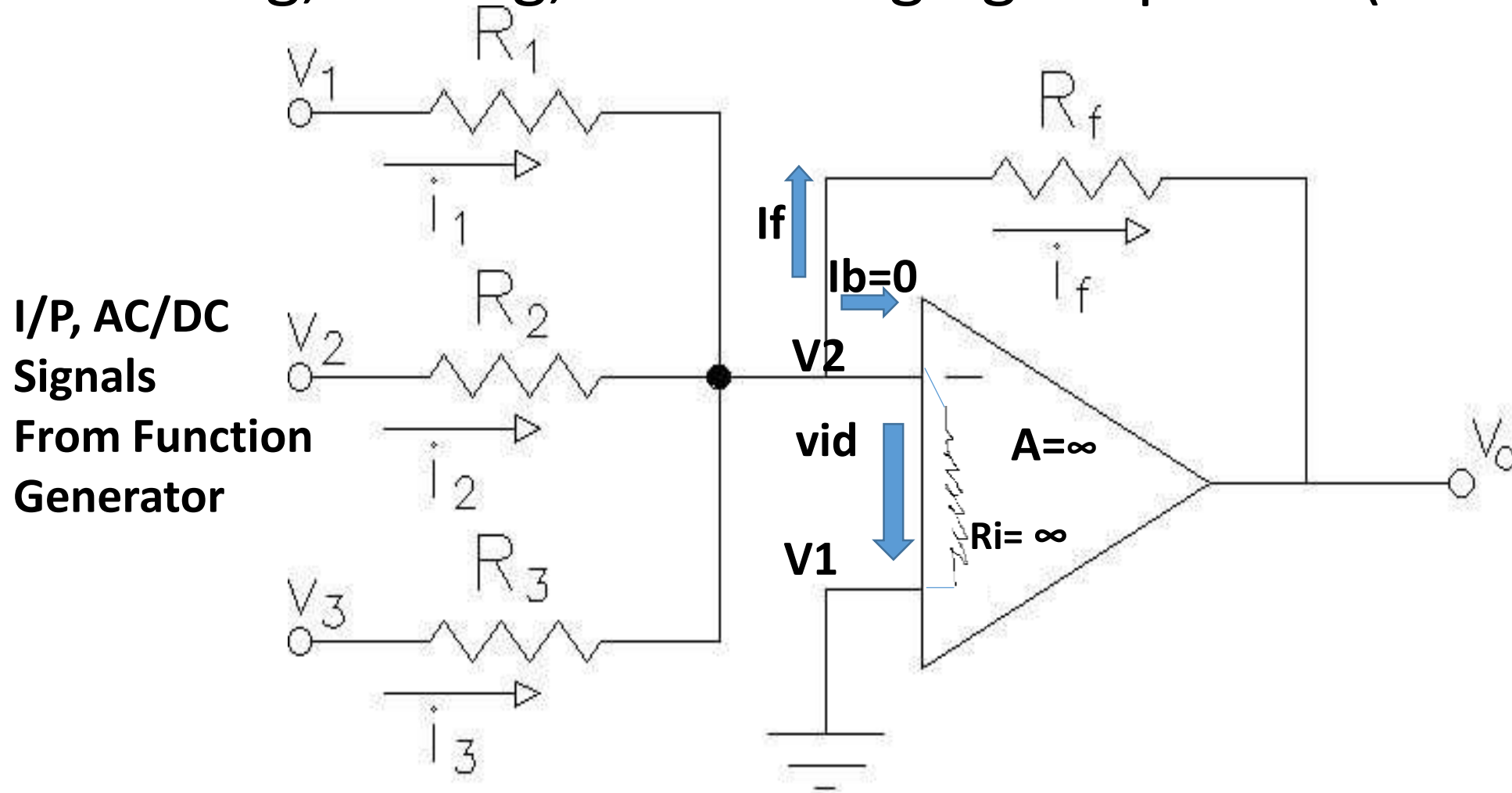
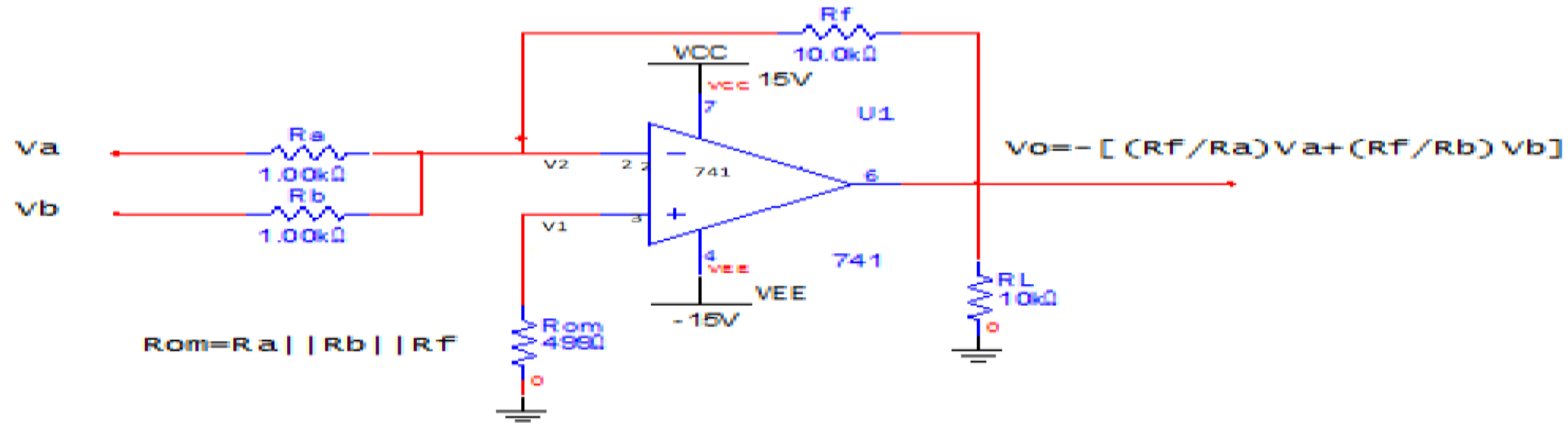
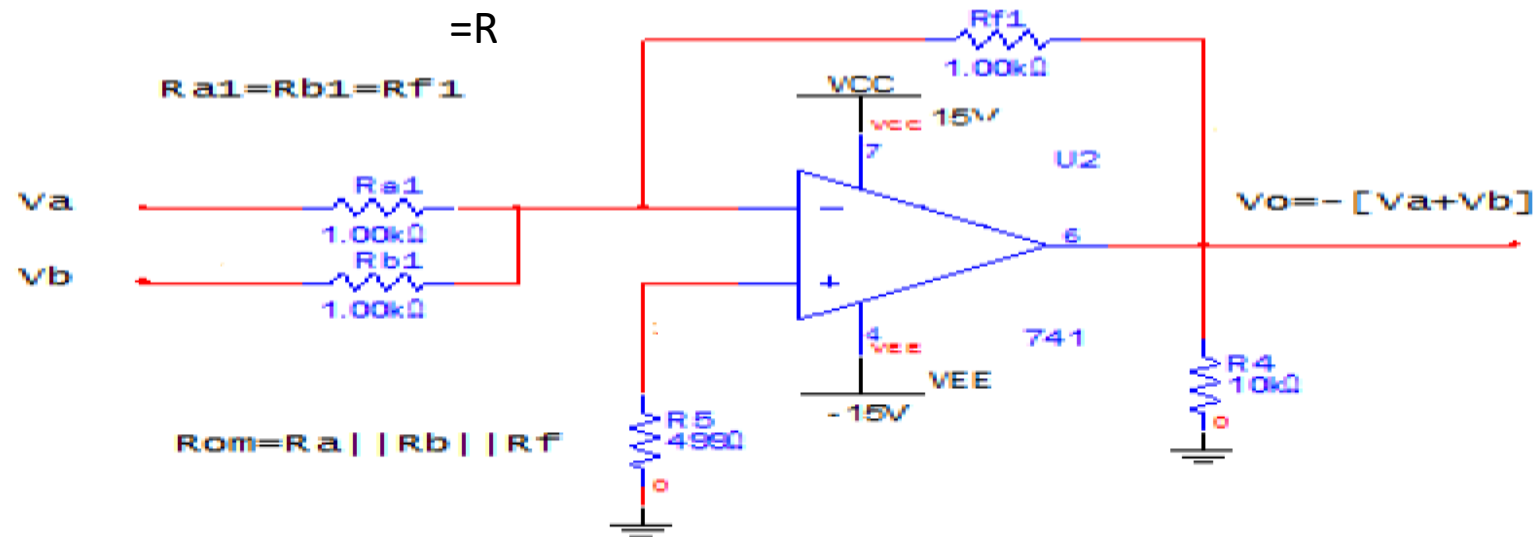


Fig. Inverting Configuration with Three inputs can be used as a Summing, Scaling, and Averaging Amplifiers

Mathematical Analysis for 2 Inputs Summing, Scaling, and Averaging Amplifiers



OP-AMP as a Summing Amplifier with inverting configuration



OP-AMP as a Summing Amplifier with Gain=1

- From above circuit by applying KCL at Node V2.
- $I_1 + I_2 = I_B + I_F$
- Since R_i and A of the OP-AMP are ideally infinity,
- Therefore, $V_1 = V_2 = 0$ V, Also $I_B = 0$ amp.
- Therefore, $I_1 + I_2 = I_F$
- $V_a/R_a + V_b/R_b = -V_o/R_f$
- So, $V_o = -R_f (V_a/R_a + V_b/R_b)$ ----- This is the imp. relation
- If $R_a = R_b = R_f = R$
- Then $V_o = -(V_a + V_b)$ Summing
- $V_o = -(R_f/R_a * V_a + R_f/R_b * V_b)$ Scaling Amplifier.
- If $R_f/R = 1/n = 1/2$, $V_o = -1/2 (V_a + V_b)$ Averaging Amplifier
- Where $n =$ no. of inputs

Example : Design an Adder circuit using an op-amp to get output expression as $V_o = - (0.1 V_a + V_b + 10 V_c)$. Draw the circuit Diag.

Where V_a , V_b , and V_c are the inputs

Solution :
$$V_o = - \left(\frac{R_f}{R_1} V_1 + \frac{R_f}{R_2} V_2 + \frac{R_f}{R_3} V_3 \right) \text{ ----- this is the std. equation}$$

Comparing Given Equation with std. equation, we get

$$R_f/R_a = 0.1 , \quad R_f/R_b = 1, \quad R_f/R_c = 10$$

Assume $R_f = 10 \text{ k}$, Find R_a , R_b , R_c

$$R_a = 100 \text{ K} , \quad R_b = 10 \text{ K} , \quad R_c = 1 \text{ k}$$

Then the desired output expression is Obtained.

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \frac{V_o}{R_f} = 0$$

or,

$$V_o = -\left(\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3\right) \quad (4)$$

Thus the output is an inverted, weighted sum of the inputs. In the special case, where $R_1 = R_2 = R_3 = R_f$, we have

$$V_o = -(V_1 + V_2 + V_3) \quad (5)$$

in which case the output V_o is the inverted sum of the input signals. We may also set

$$R_1 = R_2 = R_3 = 3R_f$$

in which case

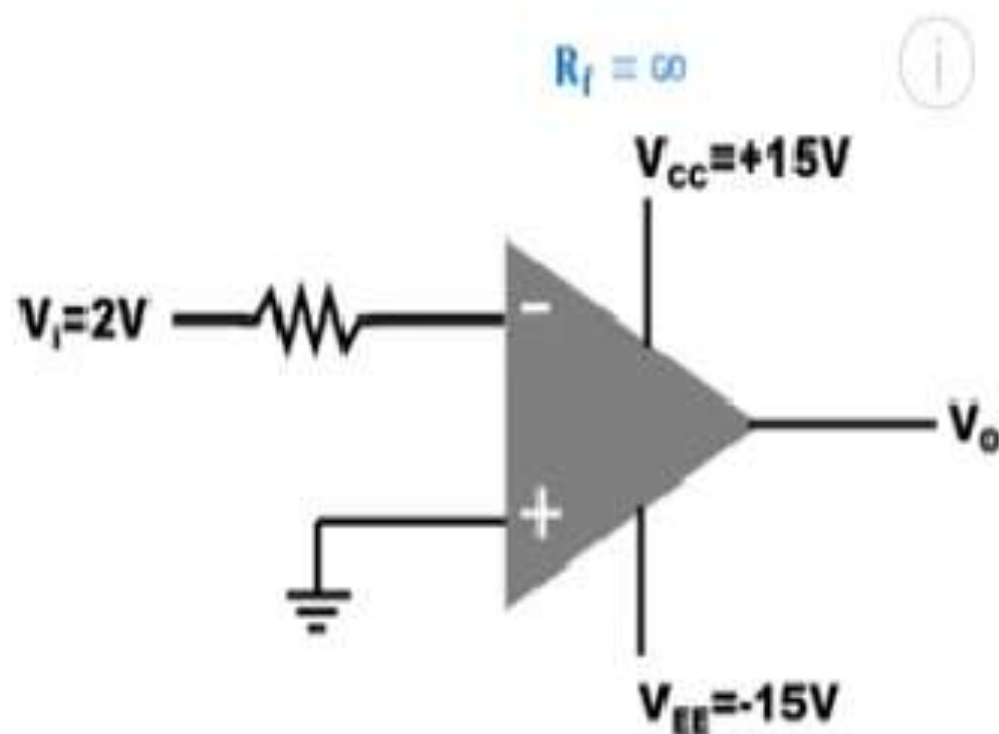
$$V_o = -\left(\frac{V_1 + V_2 + V_3}{3}\right) \quad (6)$$

Ex. 8 Determine the output voltage.

$$\text{Voltage Gain} = \frac{V_o}{V_2 - V_1} = \infty$$

Due to the Range of V_{CC} and V_{EE}

Output Voltage will be either +15V or -15V,
depends on the inverting and non - inverting Op - amp



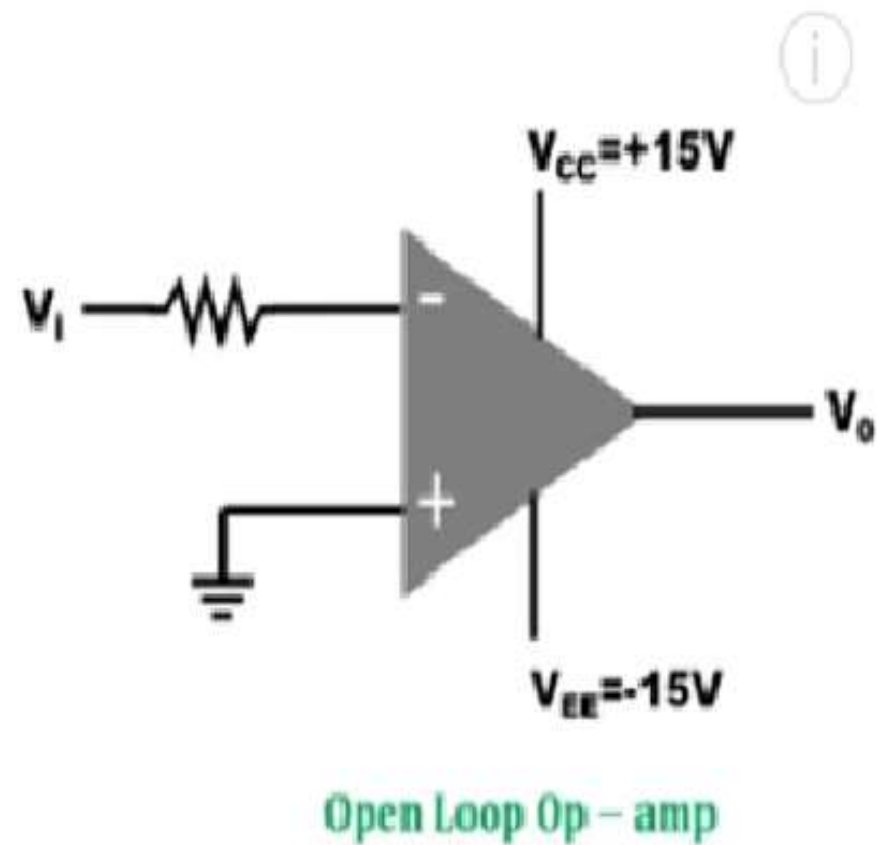
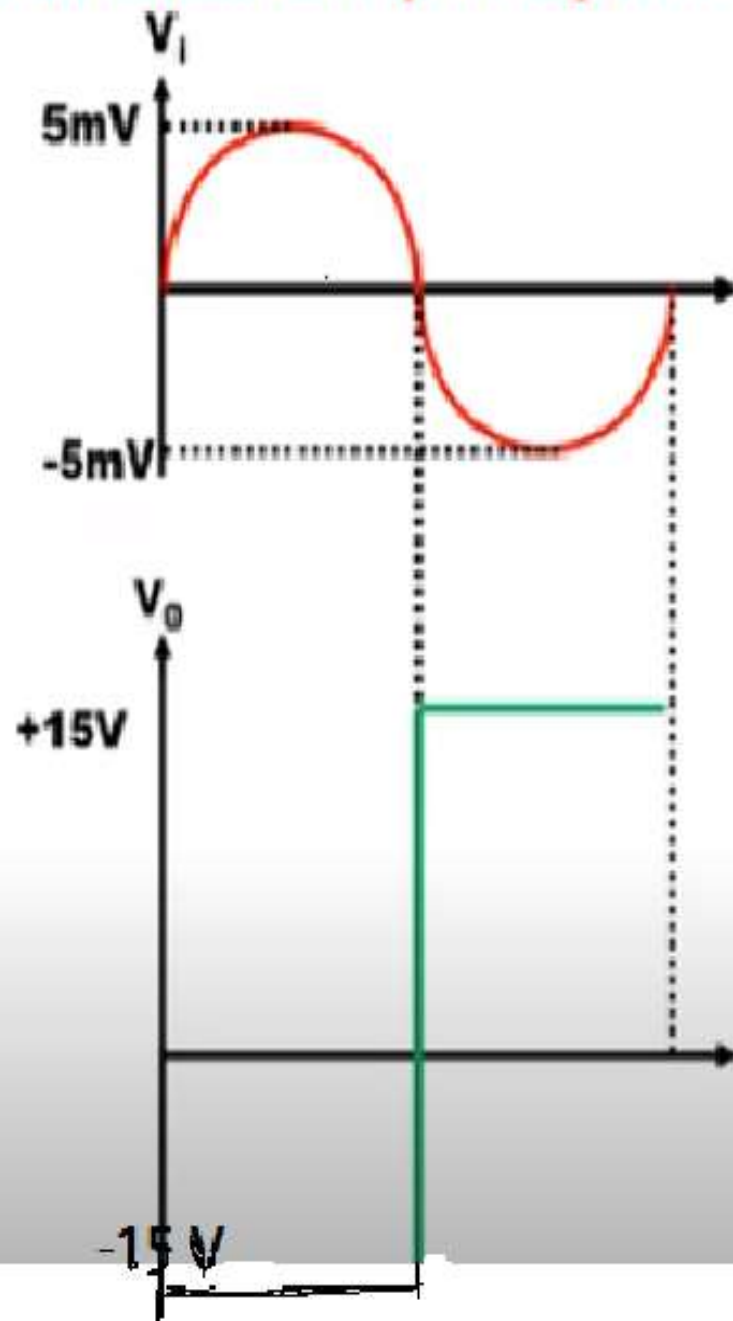
Open Loop Op - amp

$$\left(1 + \frac{R_f}{R_i}\right) = \infty \quad (\text{For Non - inverting})$$

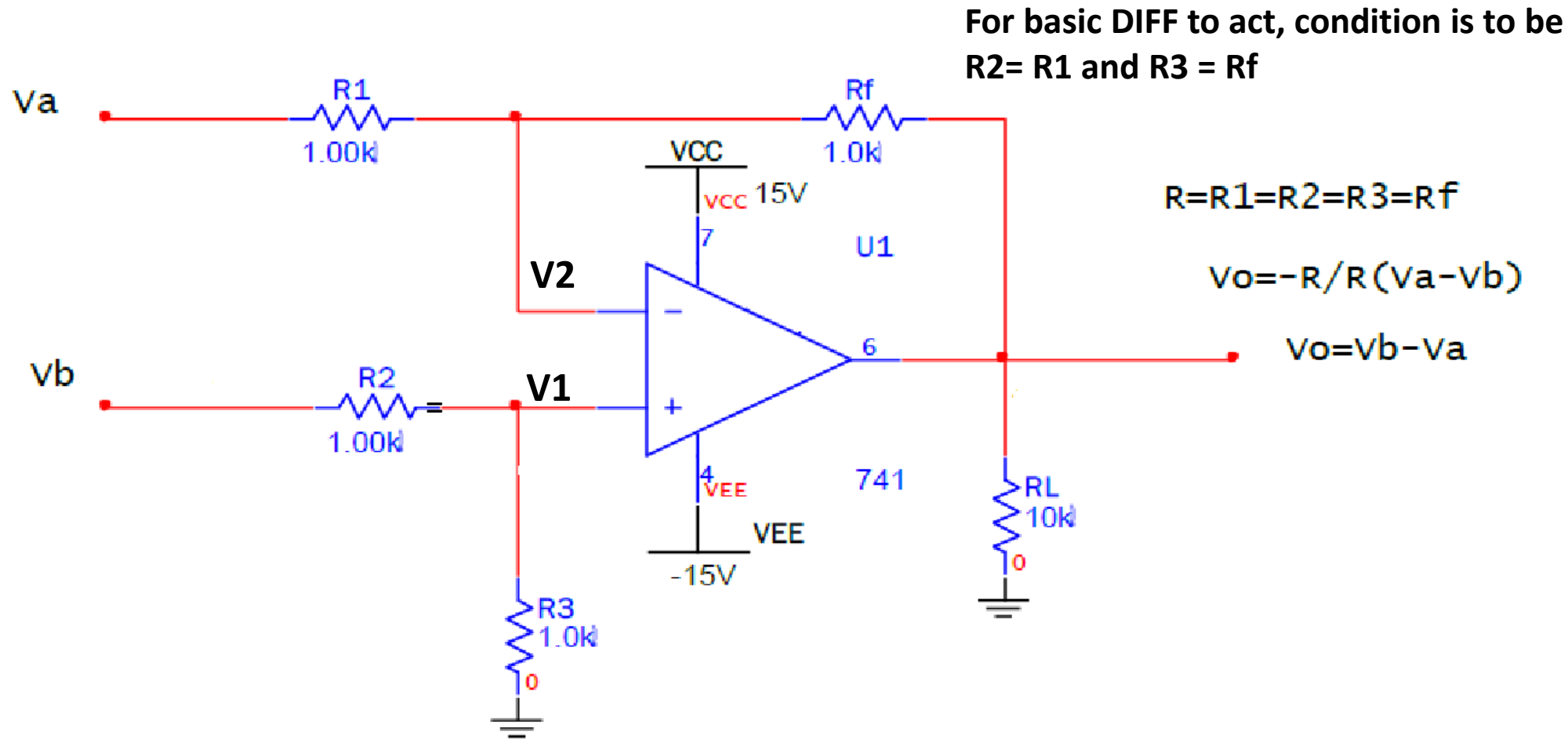
$$\left(\frac{R_f}{R_i}\right) = \infty \quad (\text{For inverting})$$



Ex. 9 Determine the output voltage curve.



Basic Differential Amplifier Circuit (using one op-amp) and also Acts as a Subtractor



Differential Amplifier as a subtractor

- Differential Amplifier is a combination of Inverting and Non-inverting amplifier.

Step 1 :- ie. When V_a is reduced to zero the circuit is a **Non-Inverting amplifier**, whereas the circuit is an **Inverting amplifier** when input V_b is reduced to zero.

- Therefore we will use the, **Superposition theorem** in order to establish relationship between Input and output. when $V_b = 0$ V, the configuration becomes an Inverting amplifier, hence the output due to V_a only is.

- $V_{oa} = -\frac{R_f}{R_1} (V_a)$ -----eq 1

- **Step 2 :** Similarly, when $V_a = 0$ V, the configuration is a Non- Inverting amplifier having a voltage- divider network composed of **R2 and R3** at the Non- Inverting input, therefore,

- $V_1 = \frac{R_3}{R_2+R_3} (V_b)$

- and the output due to V_b , then is

- $V_{ob} = (1 + \frac{R_f}{R_1}) V_1$ -----eq 2

- ie. $V_{ob} = (\frac{R_3}{R_2+R_3}) (\frac{R_1+R_f}{R_1}) V_b$

- since $R_2 = R_1$ and $R_3 = R_f$ ----- **necessary condition for the DIFF AMP**

- $V_{ob} = \frac{R_f}{R_1} V_b$ -----eq 2

- Thus from eq 1 n eq 2, the net output voltage is

- $V_o = V_{oa} + V_{ob}$

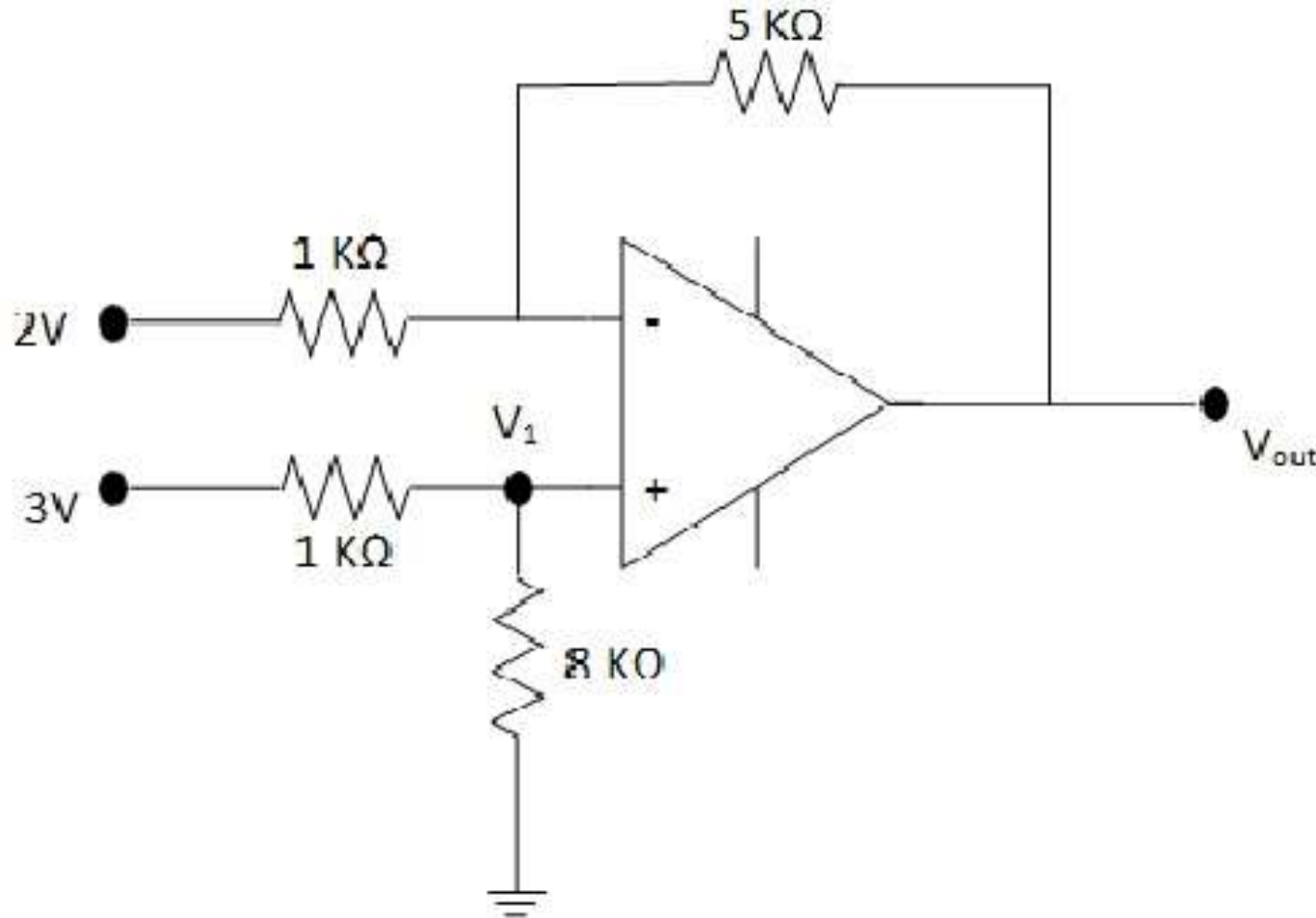
- $V_o = -\frac{R_f}{R_1} (V_a - V_b) = -\frac{R_f}{R_1} (V_{ab}) = + R_f/R_1 (V_{ba})$ -----**equation of Diff Amp with gain**

- $R_f = R_1 = R$

- $V_o = (V_b - V_a)$ ----- **further acts as a Subtractor**

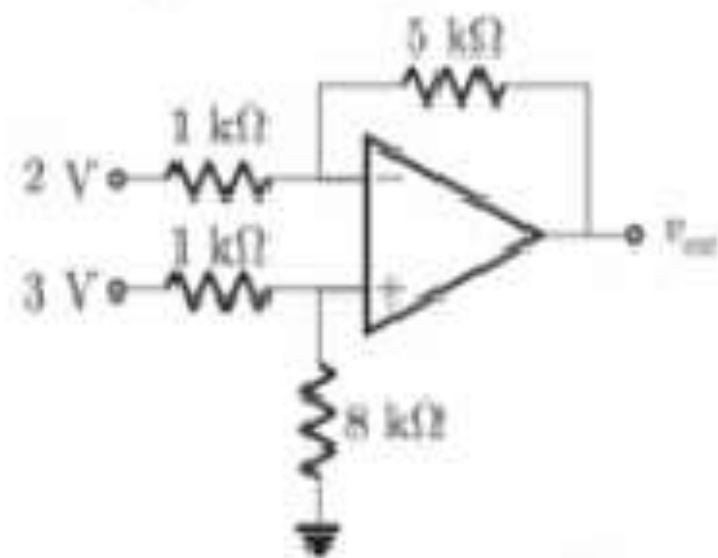
- or the voltage gain, $A_d = \frac{V_o}{V_{ab}} = -\frac{R_f}{R_1} =$ gain of DIFF AMP, which is equal to the gain of Inverting Amplifier.

Exercise 1: Find output voltage for the circuit shown



Use Superposition theorem to find V_o , ie. Consider one input at a time with other input connected to ground.

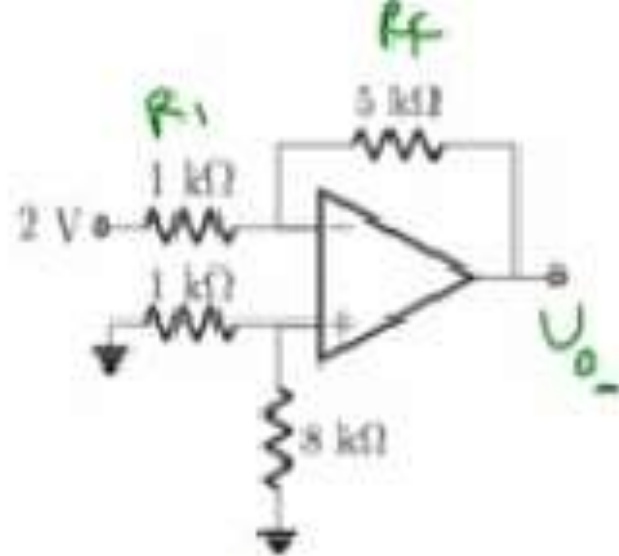
If the op-amp in the figure is ideal, the output voltage V_{out} will be



- (A) 1 V (B) 6 V
 (C) 14 V (D) 17 V

$$V_o = V_{o-} + V_{o+}$$

$$V_o = -10 + 16 = 6 \text{ volts}$$

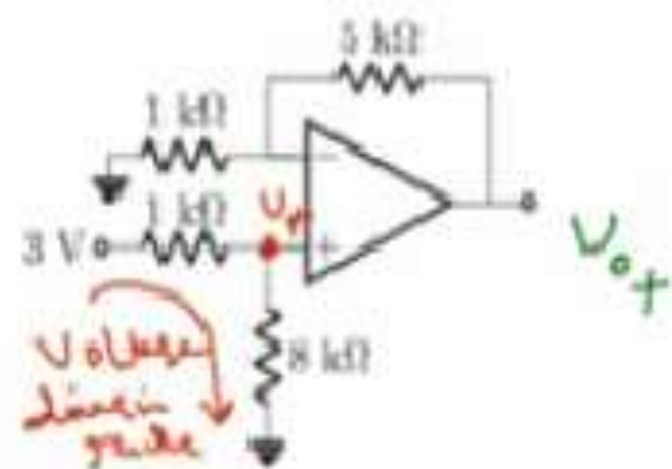


inverting op-amp

$$V_{o-} = -\frac{R_f}{R_1} \cdot V_{in}$$

$$= -\frac{5k}{1k} \cdot (2V)$$

$$V_{o-} = -10 \text{ volts}$$



NIN op-amp

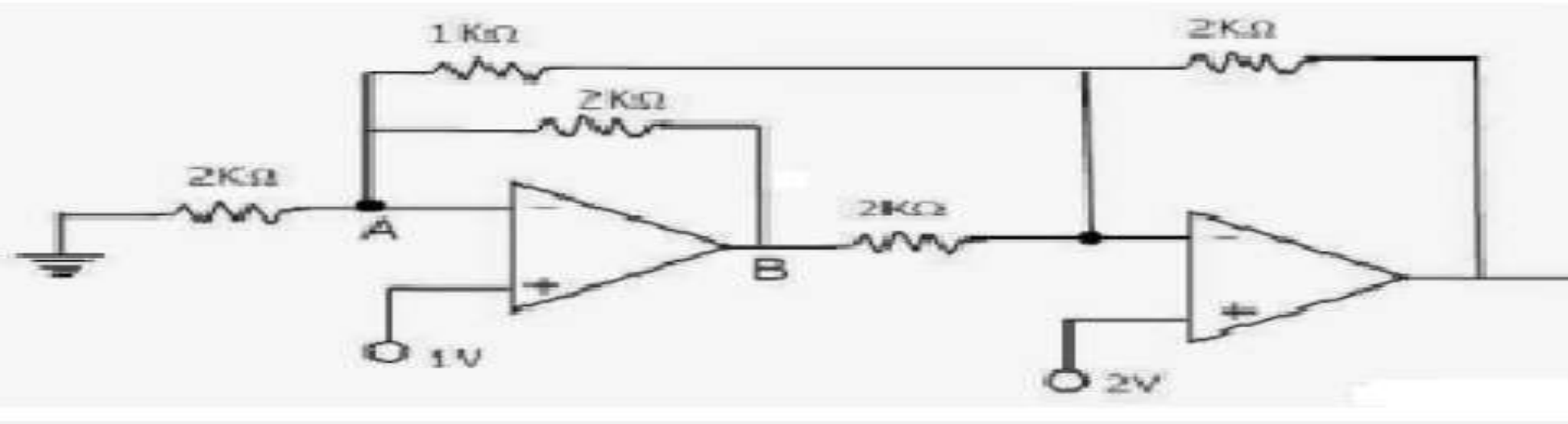
$$V_{o+} = \left(1 + \frac{R_f}{R_1}\right) U_+$$

$$= \left(1 + \frac{5}{1}\right) \left(\frac{8}{8+1}\right) \cdot 3V$$

$$= \frac{2}{6} \times \frac{8}{9} \times 3$$

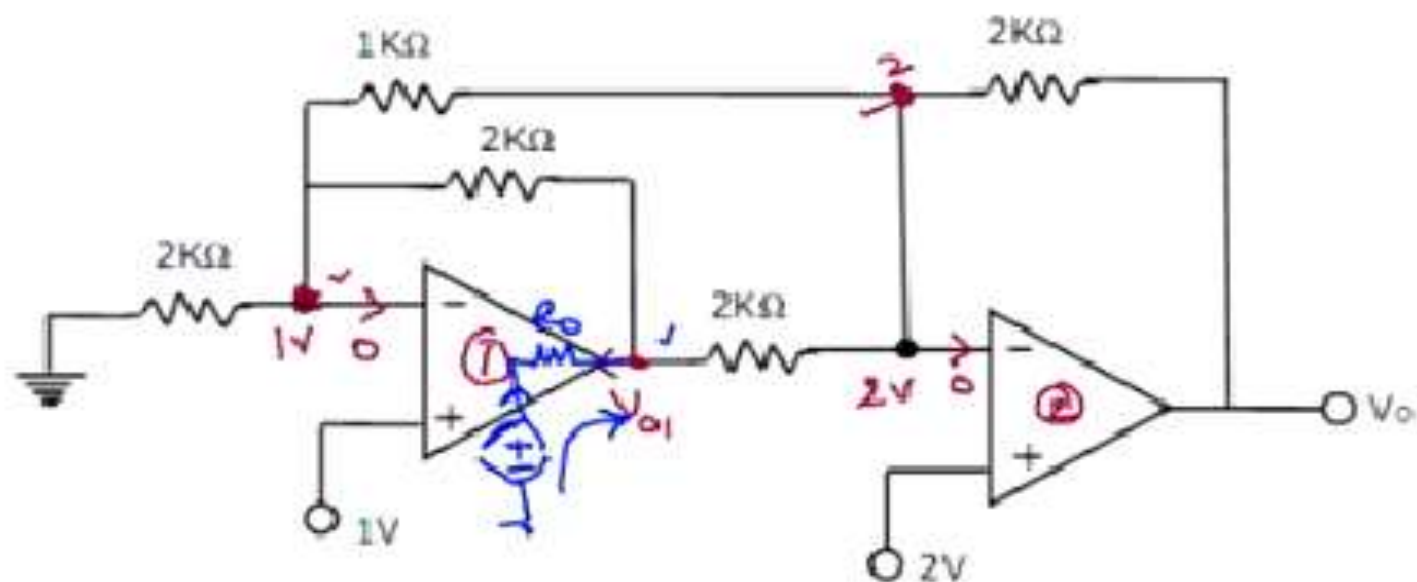
$$V_{o+} = 16 \text{ volts}$$

Exercise 2 : If the op-amp in the figure is ideal, the output voltage V_{out} will be equal to
Options a. 1 V b. 6 V c. 17 V d. 18 V GATE question



V_o

Find the output voltage, V_o in the following circuit (figure), assuming that the op-amps are ideal.



Ideal op AMP : $A_{OL} = \infty$

$$V_o = A_{OL} \cdot V_{id}$$

$$V_{id} = \frac{V_o}{A_{OL}} = \frac{V_o}{\infty} = 0$$

$$V_{id} = V_+ - V_- = 0$$

$$V_+ = V_-$$

Virtual Short

S₁: Apply KCL at 1 volt node

$$\frac{1}{2} + \frac{1 - V_{o1}}{2} + \frac{1 - 2}{1} = 0$$

$$1 + 1 - V_{o1} - 2 = 0$$

$$V_{o1} = 0 \text{ volts}$$

S₂: KCL at node 2V,

$$\frac{2 - 1}{1} + \frac{2 - V_{o1}}{2} + \frac{2 - V_o}{2} = 0$$

$$2 + 2 - \frac{V_{o1}}{2} + 2 - V_o = 0$$

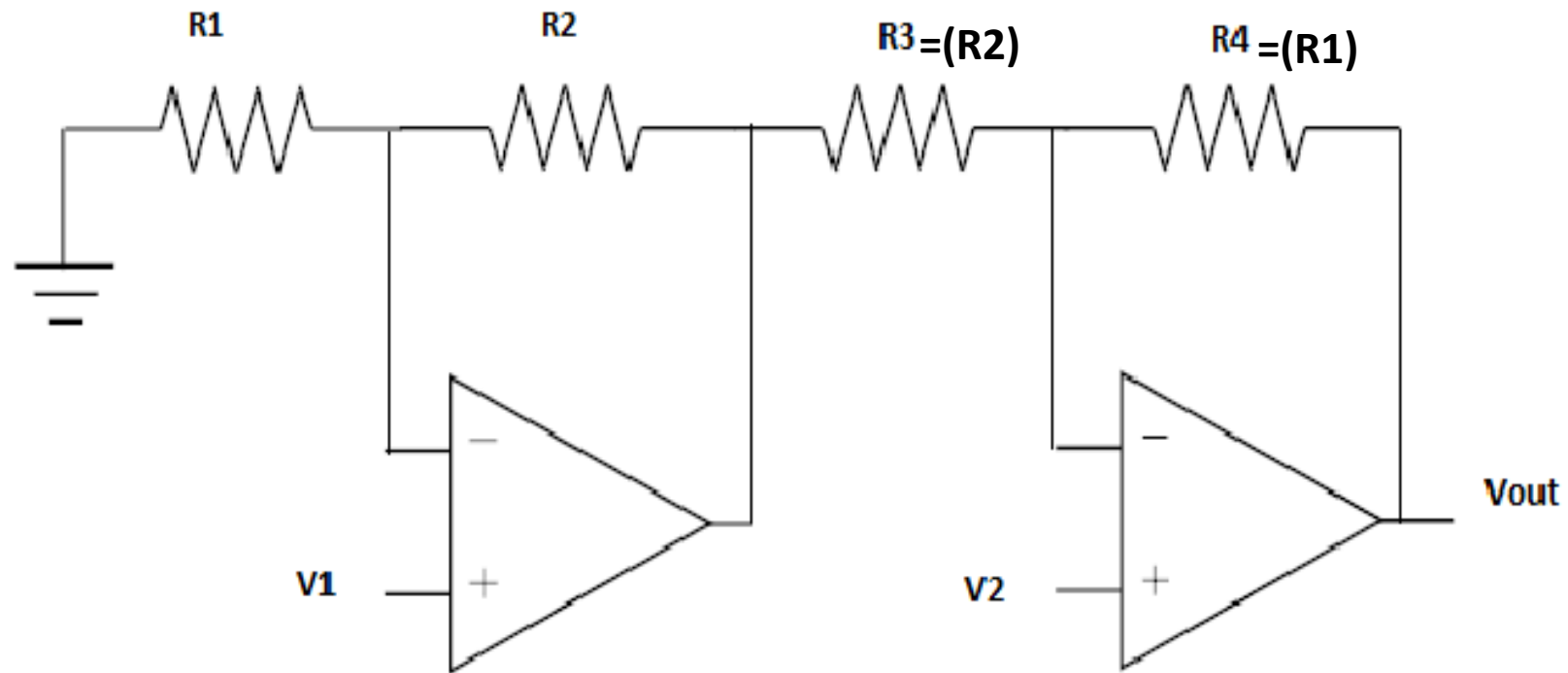
$$V_o = 6 \text{ volts}$$

KCL at V_{o1} :

$$\frac{V_{o1} - 1}{2} + \frac{V_{o1} - 2}{2} = 0$$

$$V_{o1} = 3 \text{ volts } \times$$

Two op-amp DIFF AMP



Hint:- first calculate the output voltage V_o in terms of R_1 , R_2 , R_3 , and R_4 and then substitute the condition $R_3 = R_2$ and $R_4 = R_1$ (necessary condition for DIFF AMP to act)

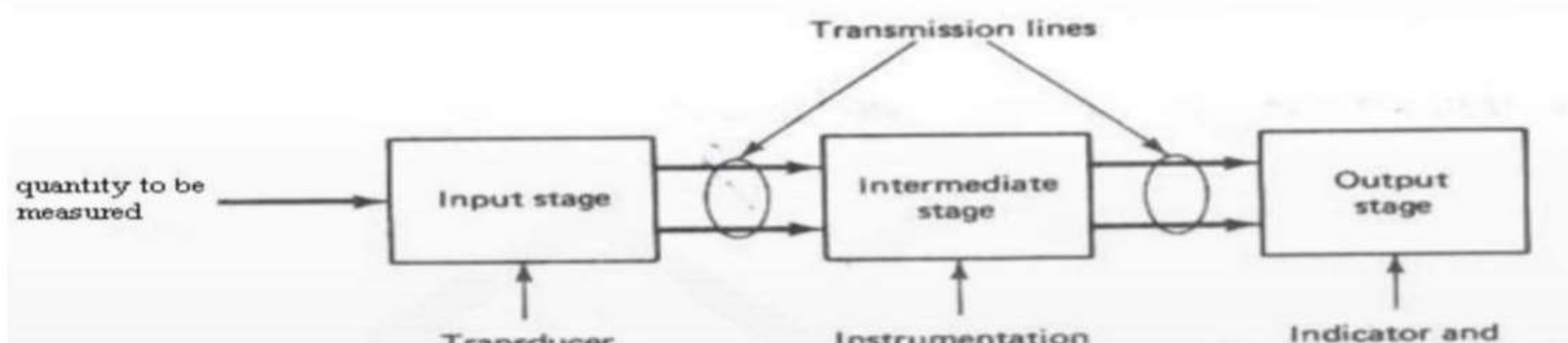
$$\text{Show that } V_o = (1 + R_2 / R_1) (V_2 - V_1)$$

Instrumentation Amplifier (using Three OP-AMP)

In a number of industrial and consumer applications, the measurement of physical quantities is usually done with the help of transducers. The output of transducer has to be amplified so that it can drive the indicator or display system. This function is performed by an instrumentation amplifier.

The important features of an instrumentation amplifier are:

- i) High gain Accuracy**
- ii) High gain Stability with Low temperature coefficient**
- iii) High CMRR**
- iv) Low dc offset**
- v) Low output impedance**
- vi) High Slew Rate**



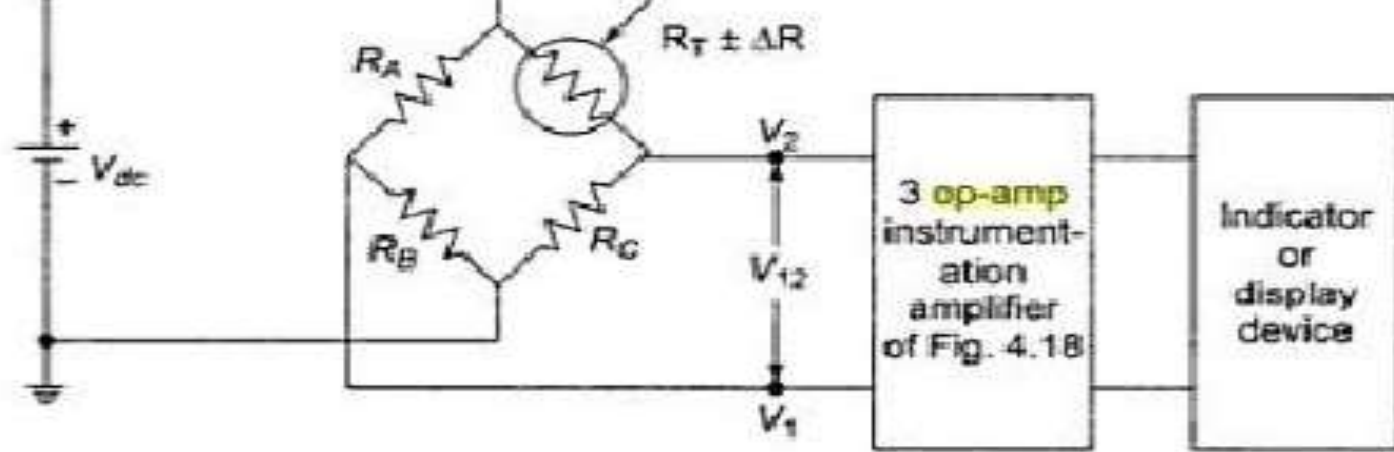
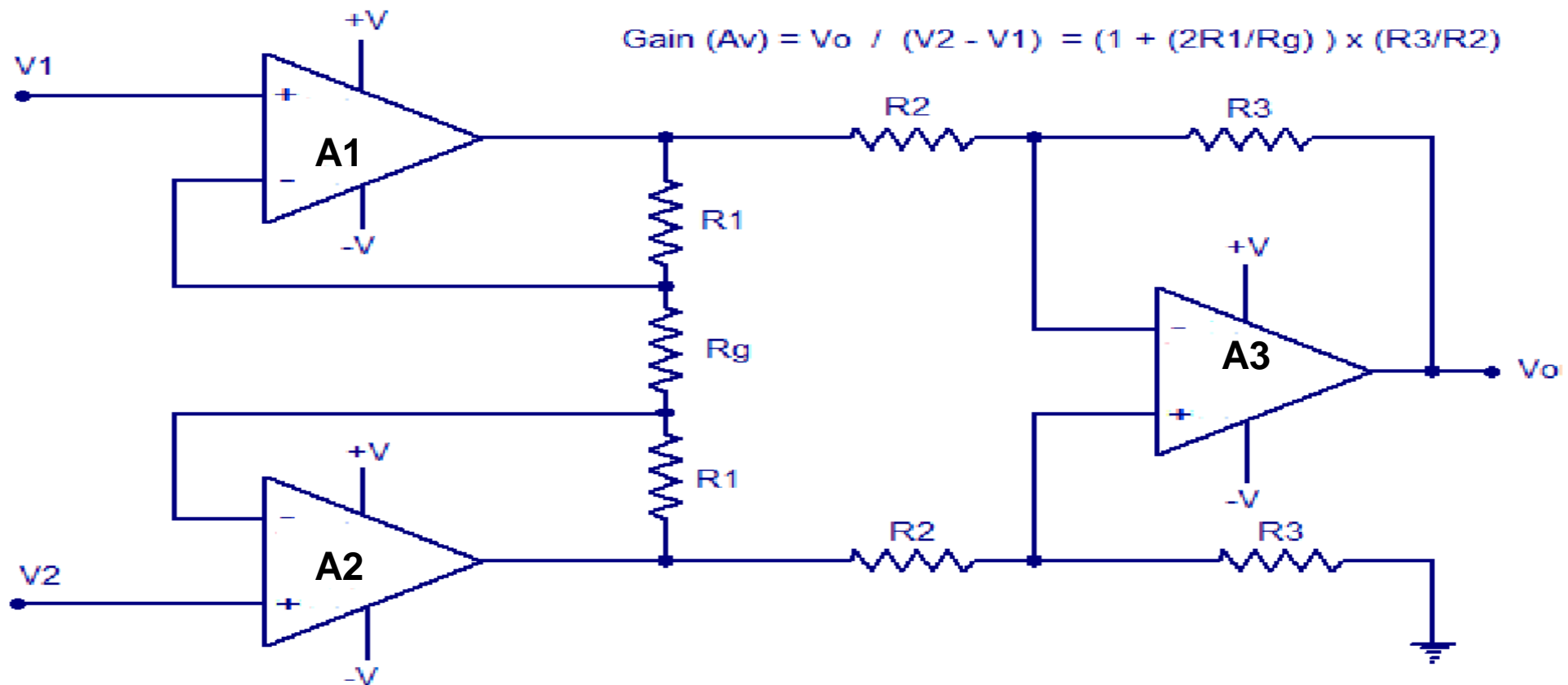
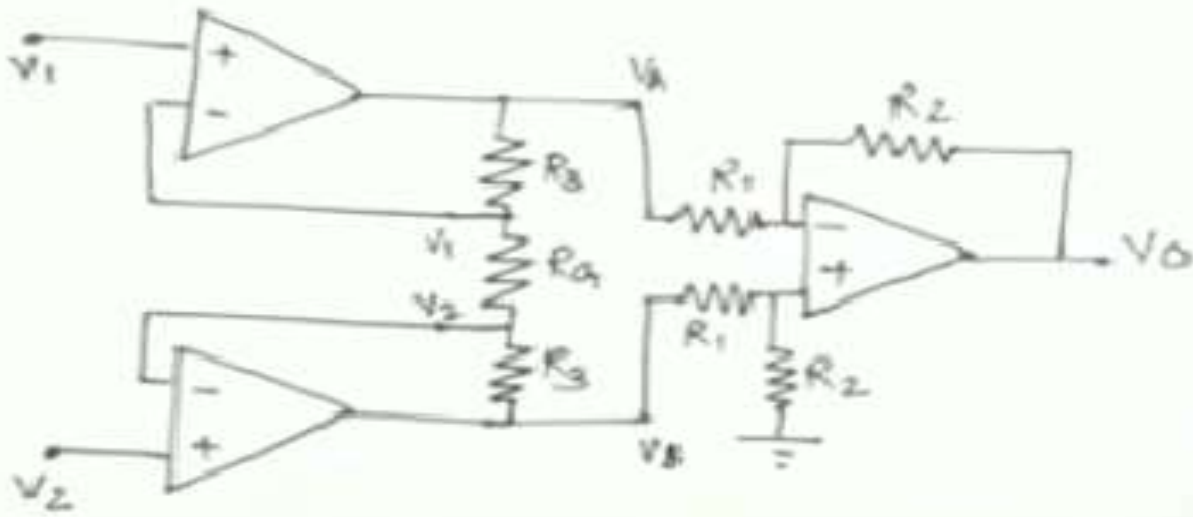


Fig.2.20 Instrumentation bridge using transducer Bridge





* $V_O = \frac{R_2}{R_1} (V_A - V_B)$... Simple difference amplifier

* I_A (current through R_A)

$$I_A = \frac{V_1 - V_2}{R_A}$$

* op-amps are ideal \rightarrow No current flowing into op-amp terminals

\therefore I_A is flowing through R_3 also.

$$I_A = \frac{V_A - V_B}{R_A + 2R_3}$$

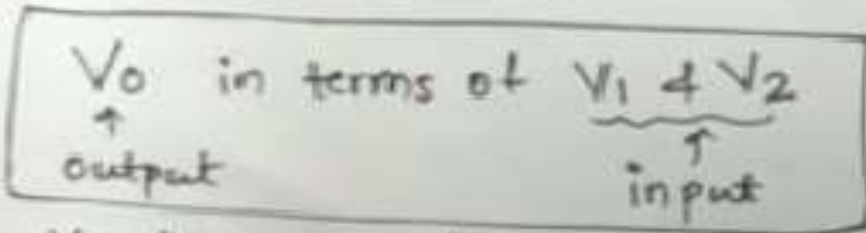
$$\therefore V_A - V_B = I_A (R_A + 2R_3)$$

Substitute I_A { in terms of V_1, V_2 }

$$* V_A - V_B = \frac{V_1 - V_2}{R_A} (R_A + 2R_3)$$

Aim :-

Output equation



V_A & V_B are intermediate voltages

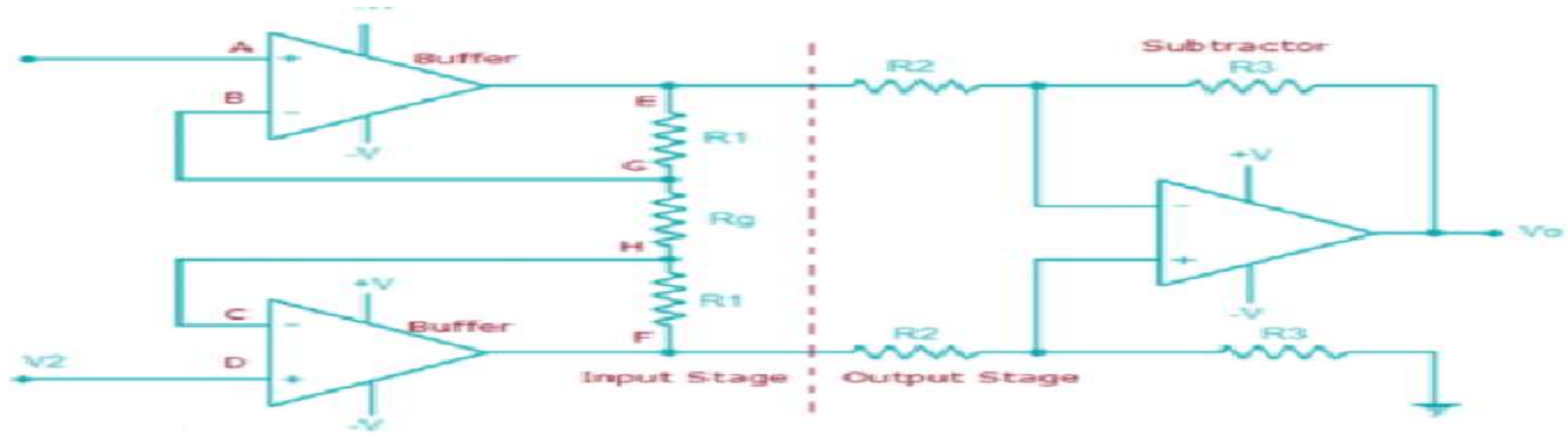
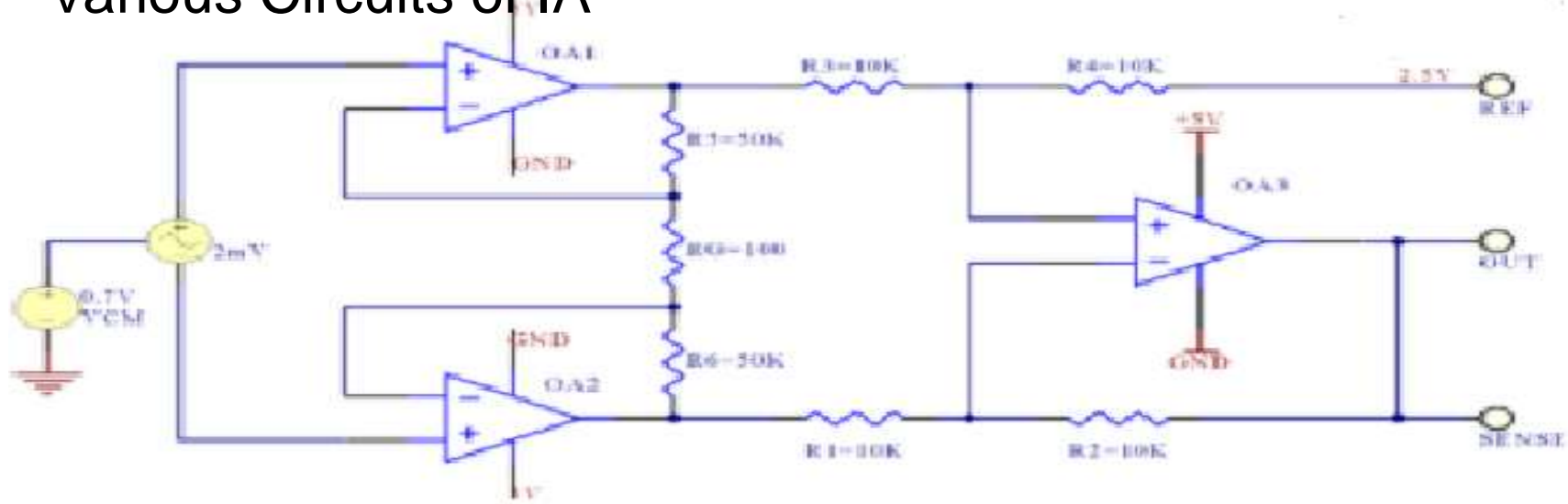
therefore we get $(V_A - V_B)$, so that, $V_O = (R_2/R_1) \cdot (V_A - V_B)$

$$V_O = (R_2/R_1) \cdot (1 + 2R_2/R_G) \cdot (V_1 - V_2)$$

Where, (R_2/R_1) is Gain of 2nd stage, and $(1 + 2R_2/R_G)$ is Gain of first stage

Output voltage = overall gain * Input Difference

Various Circuits of IA



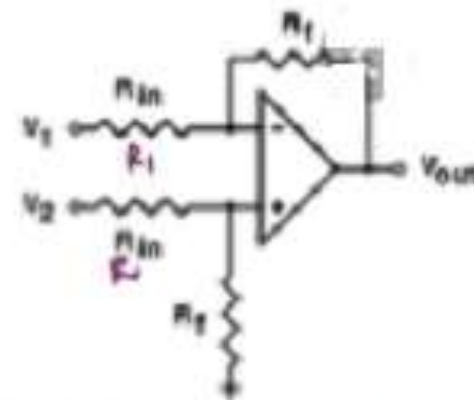
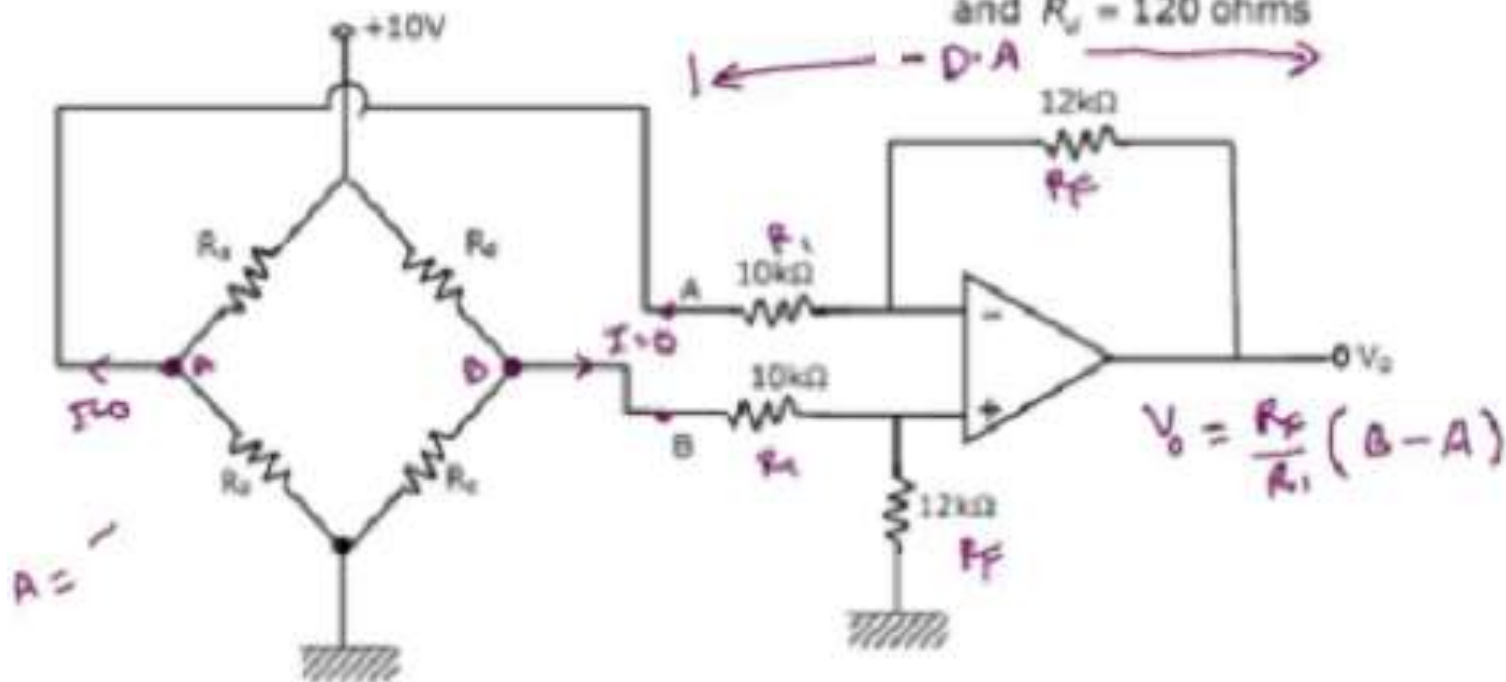
GATE 1992 (SM Question-)

Consider the circuit shown in figure. This circuit uses an ideal operational amplifier. Assuming that the impedances at nodes A and B do not load the preceding bridge circuit, calculate the output voltage V_o .

(a) when $R_1 = R_2 = R_3 = R_4 = 100 \text{ ohms}$

(b) when $R_1 = R_2 = R_3 = 100 \text{ ohms}$

and $R_4 = 120 \text{ ohms}$



$$V_o = \frac{R_f}{R_i} (V_2 - V_1)$$

D.A. Stands for Differential Amplifier

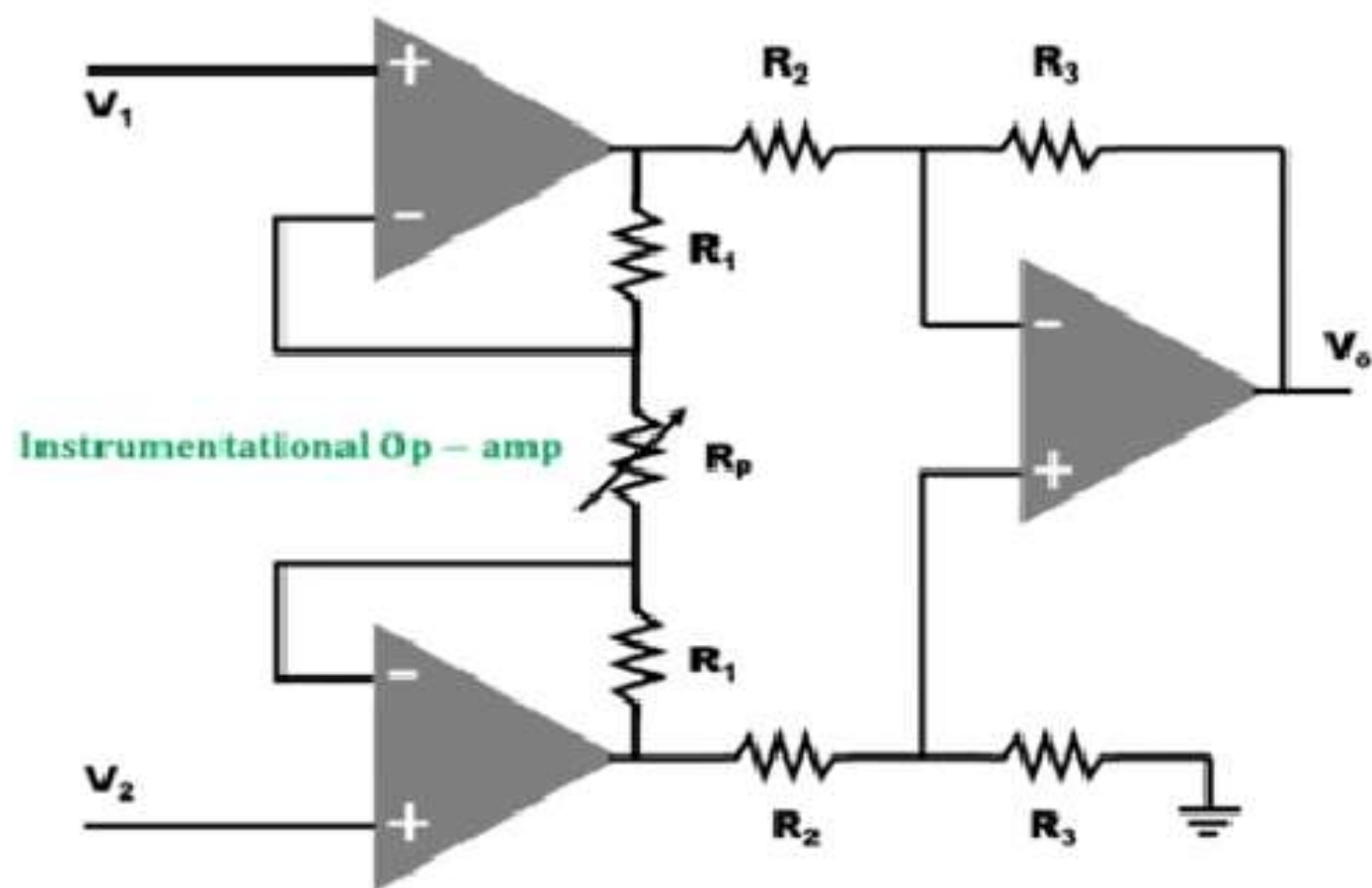
Ex. 7 Determine the output voltage, if $V_1 = 3V$, $V_2 = 1V$, $R_1 = 5K\Omega$, $R_2 = 10K\Omega$, $R_3 = 10K\Omega$ and $R_P = 5K\Omega$.

Output Voltage $\propto (V_2 - V_1)$

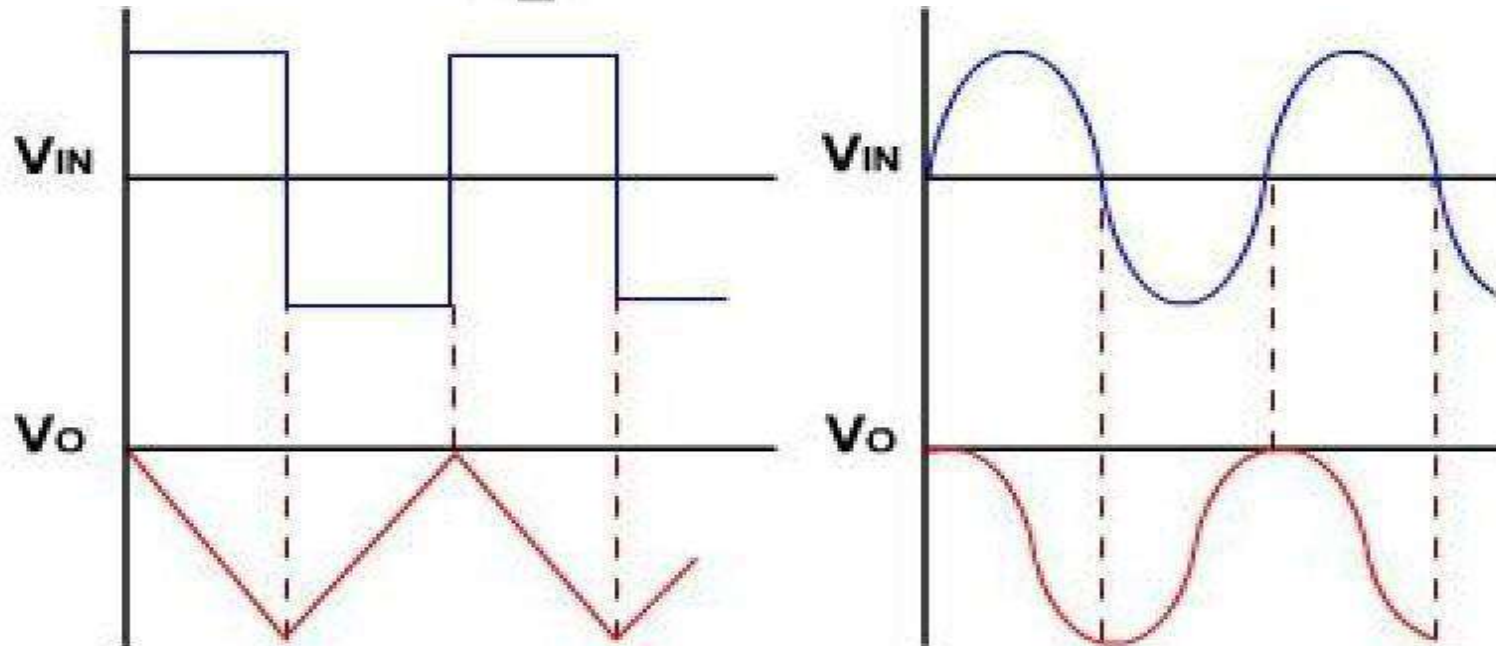
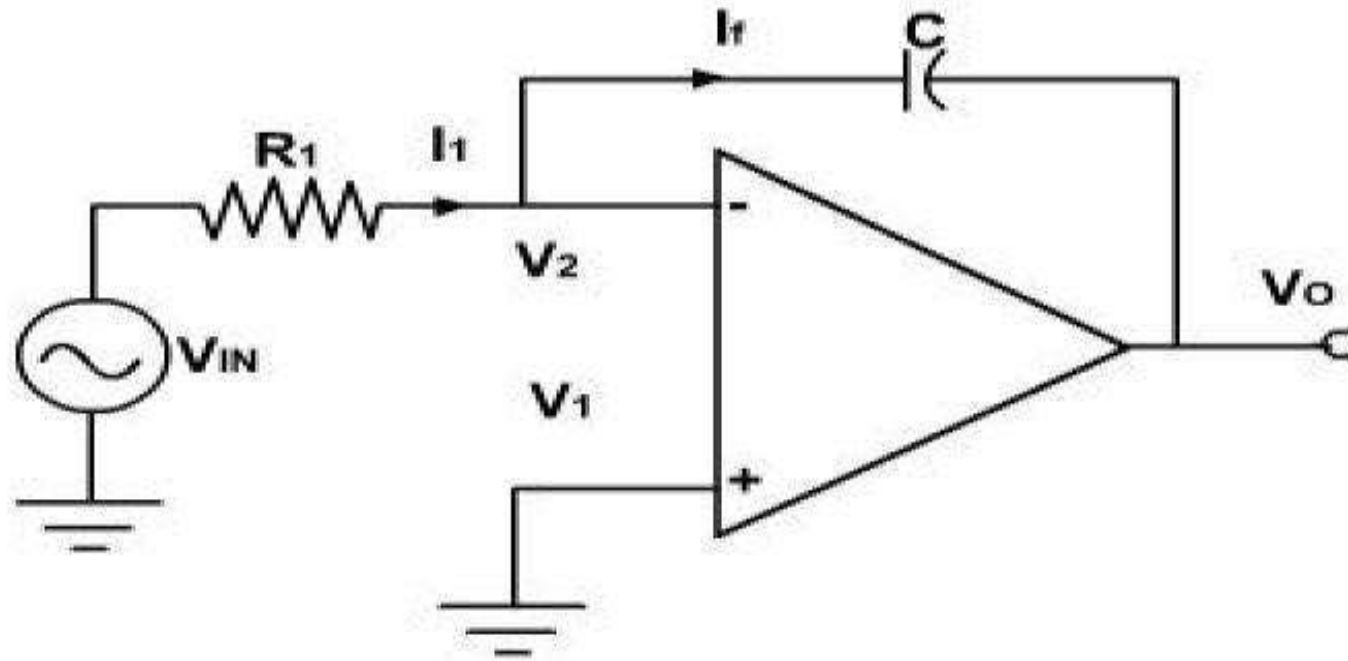
$$V_0 = (V_2 - V_1) \left(1 + \frac{2R_1}{R_P} \right) \left(\frac{R_3}{R_2} \right)$$

$$V_0 = (1 - 3) \left(1 + \frac{2 \times 5}{5} \right) \left(\frac{10}{10} \right)$$

$$V_0 = -6V$$



Basic Integrator



The Expression for output voltage V_o can be obtained by writing KCL equation at node V_1 .

$$I_i = I_B + I_F$$

$$I_i = I_f \quad \text{----- (Since } I_B = 0)$$

So, $(V_{in} - V_2) / R_1 = C_F \cdot d/dt (-V_o)$ -----($V_1 = V_2 = 0$ because A is very large)
 Integrating both sides with respect to time

$$\int (V_{in} / R_1) dt = \int C_F \cdot d/dt(-V_o) dt$$

$$= C_F (-V_o) + (V_o) |_{t=0}$$

So, rearranging the above equation

$$V_o(t) = -1 / R_1 C_F \int V_{in} dt + C$$

$$V_o(t) = -1 / R_1 C_F \int V_{in} dt \quad \text{-----as } C = V_o(0) = 0V$$

Where C is the integration constant and is proportional to the value of the output voltage V_o at time $t = 0$ seconds.

For perfect integration $T \geq R_1 C_F$ where $T =$ time period of the input signal and

$R_1 C_F = \tau =$ time constant of the circuit with which the capacitor charges & discharge.

Drawbacks of ideal integrator:

1. Bandwidth is very small and used for only small range of input frequencies.
2. For dc input ($f = 0$), the reactance of the capacitance, X_c , is infinite. Because of this op-amp goes into open loop configuration. In open loop configuration the gain is infinite and hence the small input offset voltages are also amplified and appears at output as error. This is referred as false triggering and must be avoided.

:- Due to all such limitations, an ideal integrator needs to be modified.

:- Some additional components are used along with ideal integrator circuit to reduce the effect of an error voltage in practice.

This modified integrator is referred as “**Practical Integrator**”.

Practical Integrator (lossy integrator)

- 1) The gain of an integrator at low frequency can be limited to avoid the saturation problem, therefore to avoid saturation of the op amp the feedback capacitor is shunted by a resistor R_f .
- 2) The parallel combination of R_f and C_f behaves like a practical capacitor which dissipates power, unlike an ideal capacitor.
- 3) For this reason this circuit is also called a lossy integrator. The resistor R_f limits the low frequency gain to $(-R_f/R)$, generally $[R_f=10 \cdot R_1]$ and thus provides DC stabilisation.

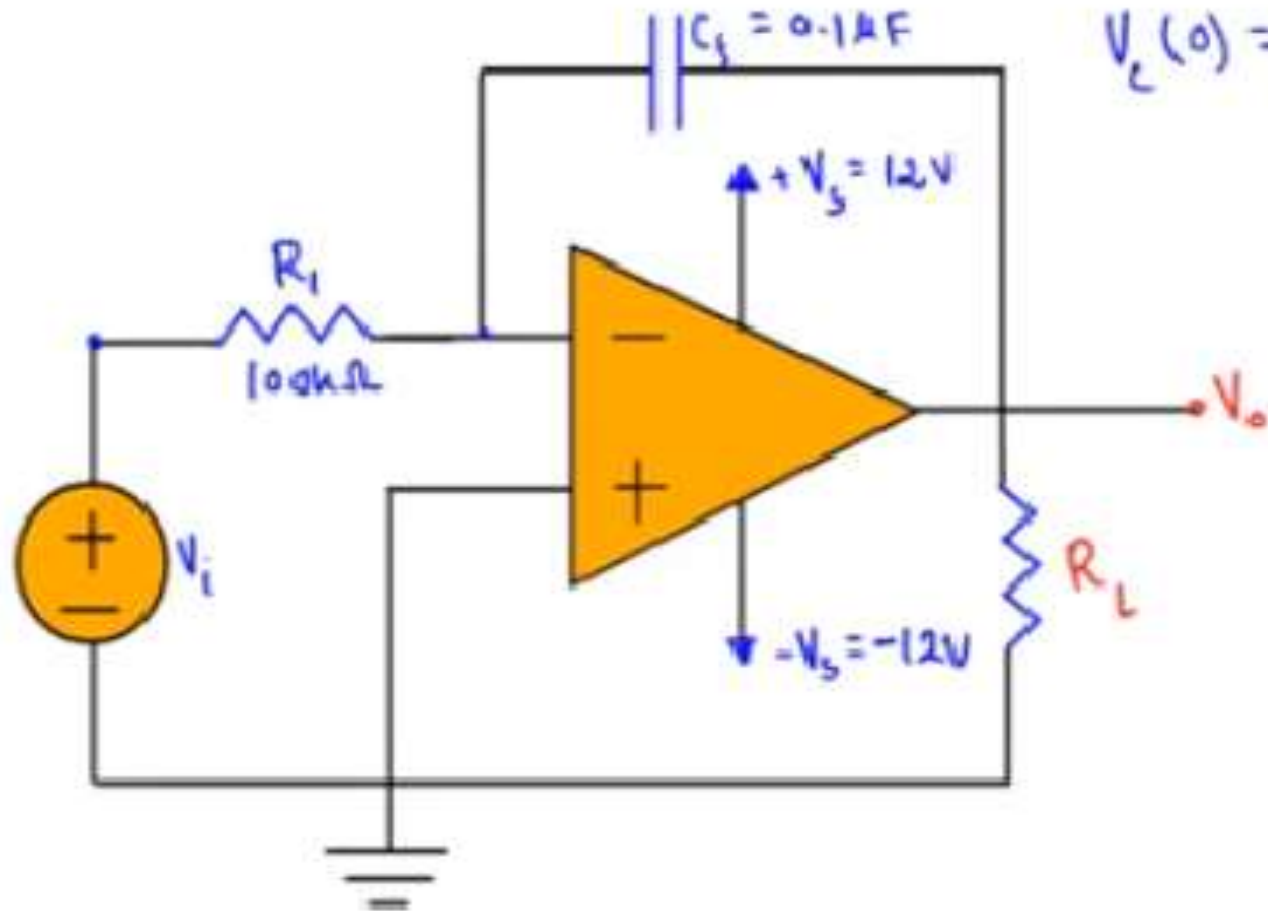
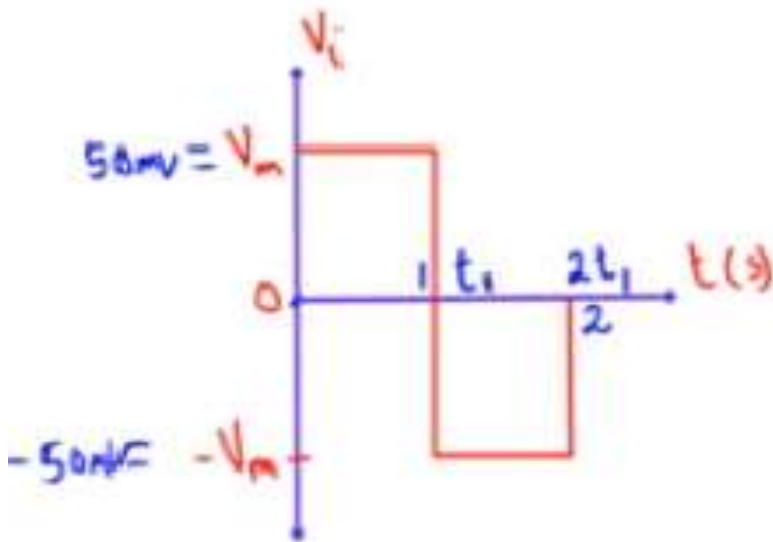
Applications:

- 1) Op-amp integrating amplifiers are used to perform calculus operations in analogue computers.
- 2) Integrating circuits are most commonly used in analogue-to-digital converters, ramp generators and also in wave shaping applications.
- 3) Another application would be to integrate a signal representing water flow, producing a signal representing the total quantity of water that has passed by the flow meter. This application of an integrator is sometimes called a totalizer in the industrial instrumentation trade.

Op-Amp Integrator : Example

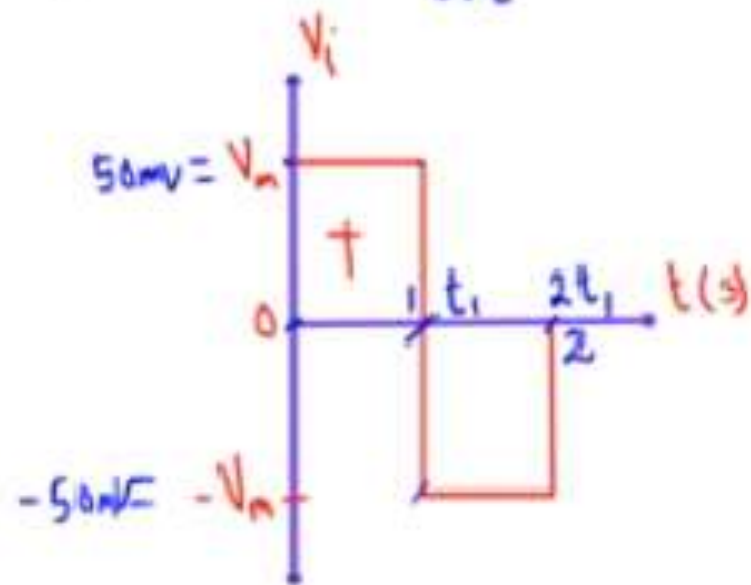
Assume the numerical values for the input voltage are $V_m = 50 \text{ mV}$, and $t_1 = 1 \text{ s}$. This input voltage is applied to the integrator amplifier. The circuit parameters of the amplifier are $R_1 = 100 \text{ k}\Omega$, $C_f = 0.1 \text{ }\mu\text{F}$, and $V_s = 12 \text{ V}$. The initial voltage on capacitor is zero. Calculate

- the output voltage $V_o(t)$
- plot $V_o(t)$ vs t .



$$V_c(0) = 0 = V_o(0)$$

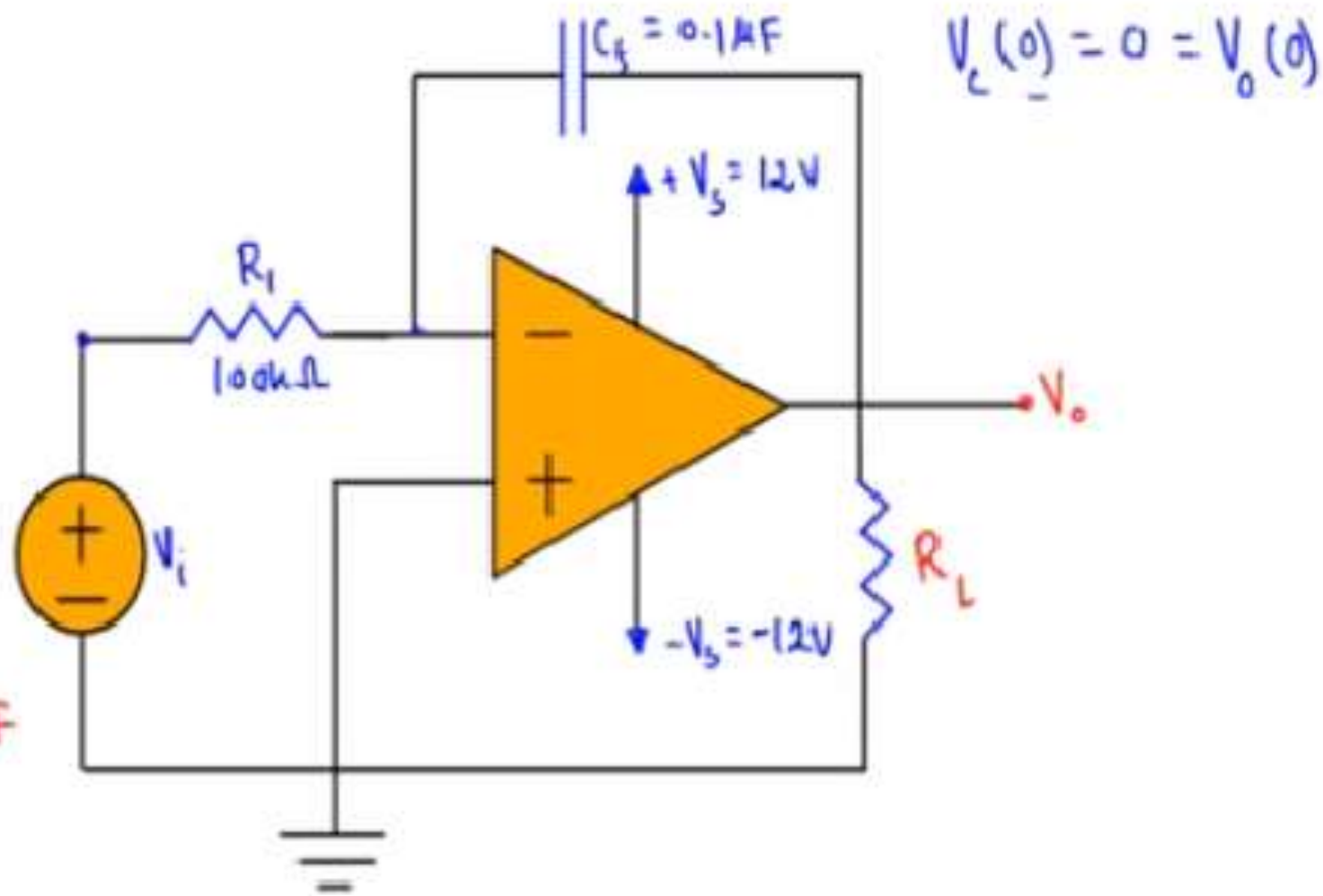
- a) the output voltage $V_o(t)$
 b) plot $V_o(t)$ vs t .



Solution: $R_1 = 100\text{k}\Omega$, $C_f = 0.1\mu\text{F}$

$$V_c(0) = 0 = V_o(0)$$

$$V_i = \begin{cases} 50\text{mV} & \text{for } 0 \leq t < 1 \\ -50\text{mV} & 1 \leq t < 2 \end{cases}$$



$$V_i = \begin{cases} 50 \text{ mV} & \text{for } 0 \leq t < 1 \\ -50 \text{ mV} & \text{for } 1 \leq t < 2 \end{cases}$$

$$V_o(t) - V_o(t_i) = -\frac{1}{R_1 C_f} \int_0^t V_i d\tau$$

For $0 \leq t < 1$, $V_o(0) = 0$

$$V_o(t) - V_o(0) = -\frac{1}{R_1 C_f} \int_0^t V_i dt = -\frac{1}{100 \times 10^3 \times 0.1 \times 10^{-6}} \int_0^t (50 \times 10^{-3}) dt$$

$$\Rightarrow V_o(t) = -5t \quad \text{for } 0 \leq t < 1$$

When $t = t_1 = 1 \text{ s}$, $V_o(t_1) = V_o(1) = -5 \times 1 = -5 \text{ V}$ [initial condition]

For $1 \leq t < 2$, $V_o(t_1) = -5V$,

$$V_o(t) - V_o(t_1) = -\frac{1}{R_1 C_f} \int_{t_1}^t V_i dt$$

$$\Rightarrow V_o(t) = -\frac{1}{100k \times 0.1 \times 10^{-6}} \int_1^t (-50 \times 10^{-3}) dt - 5$$

$$= 5 \left[t \right]_1^t - 5 = 5t - 5 - 5$$

$$\therefore V_o(t) = 5t - 10 \quad \text{for } 1 < t < 2$$

When $t = 2s$, $V_o(2) = 5 \times 2 - 10 = 0V$

$$V_o = \left\{ \begin{array}{ll} -5t & \text{for } 0 \leq t < 1 \\ 5t - 10 & \text{for } 1 \leq t < 2 \end{array} \right\}$$

st. line

$$V_o(0) = 0V$$

$$V_o(1) = -5V$$

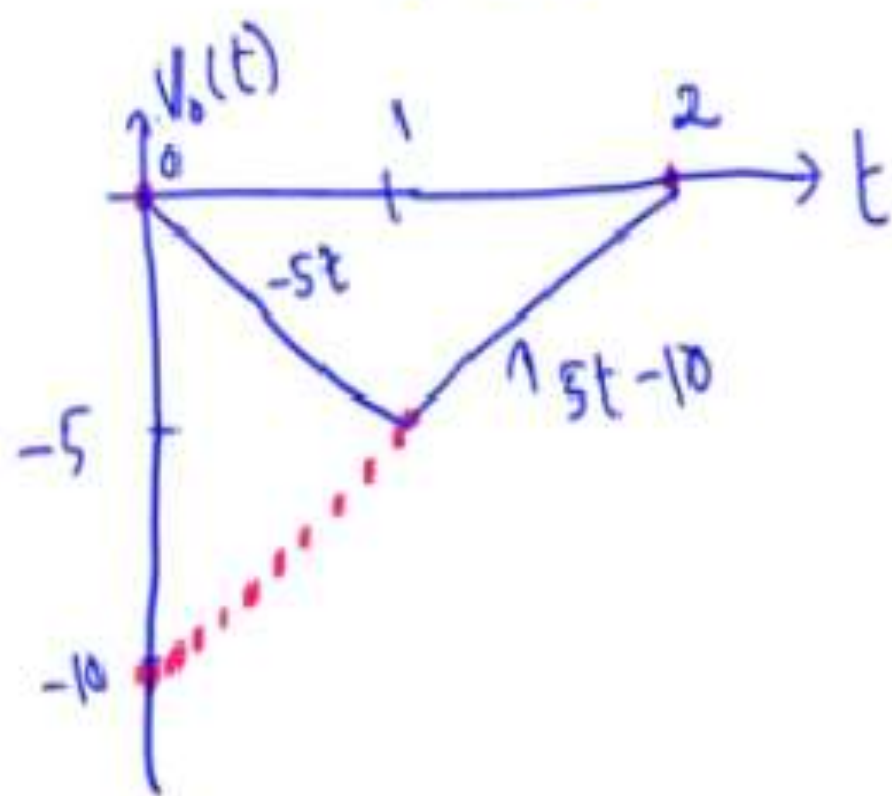
$$V_o(2) = 0V$$

$$V_o = -5t$$

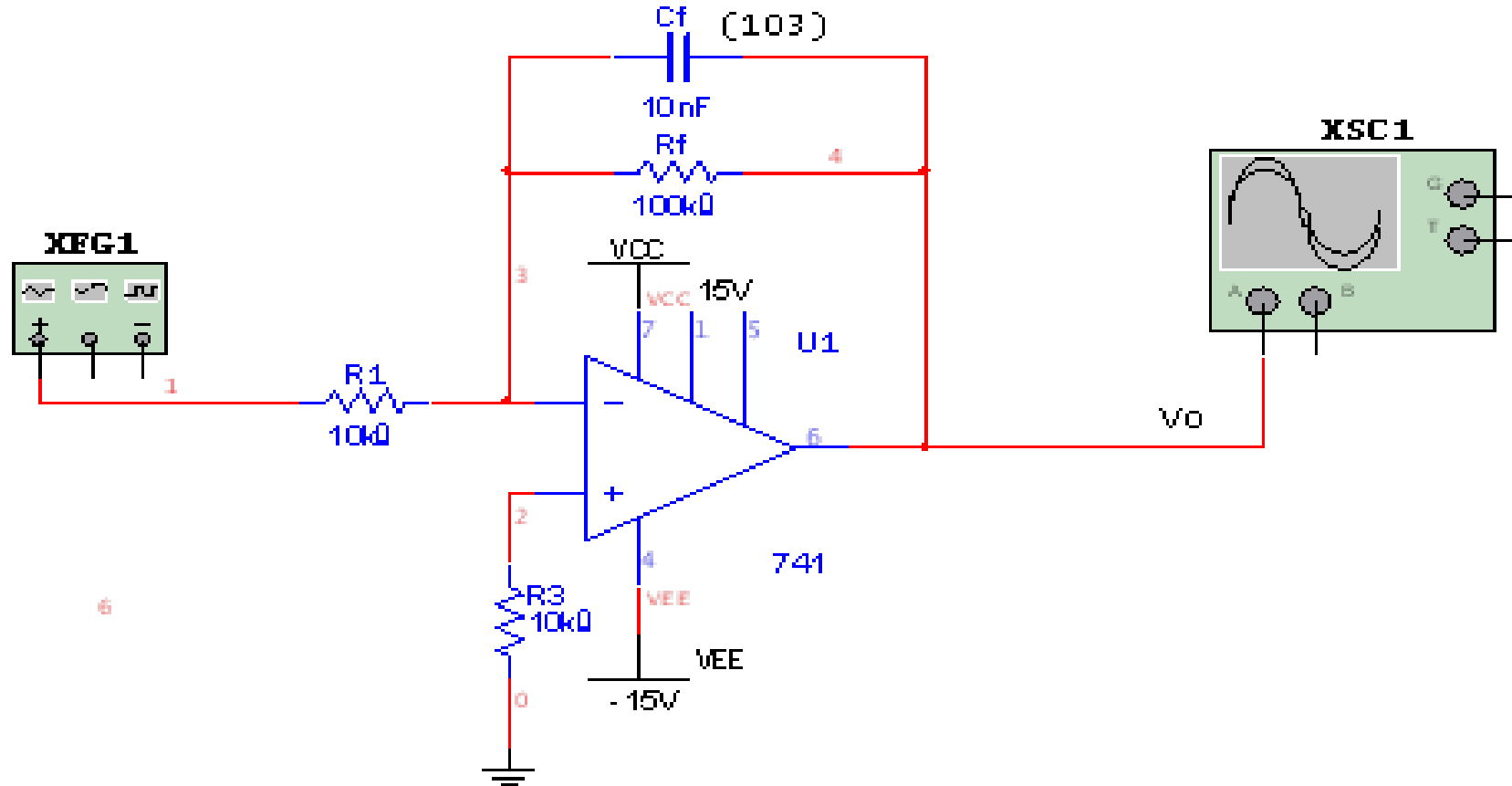
\nearrow
 $y = mx$
 $\downarrow m = -5$

$$y = mx + c$$

\nearrow \nearrow \searrow
 $V_o = 5t - 10$

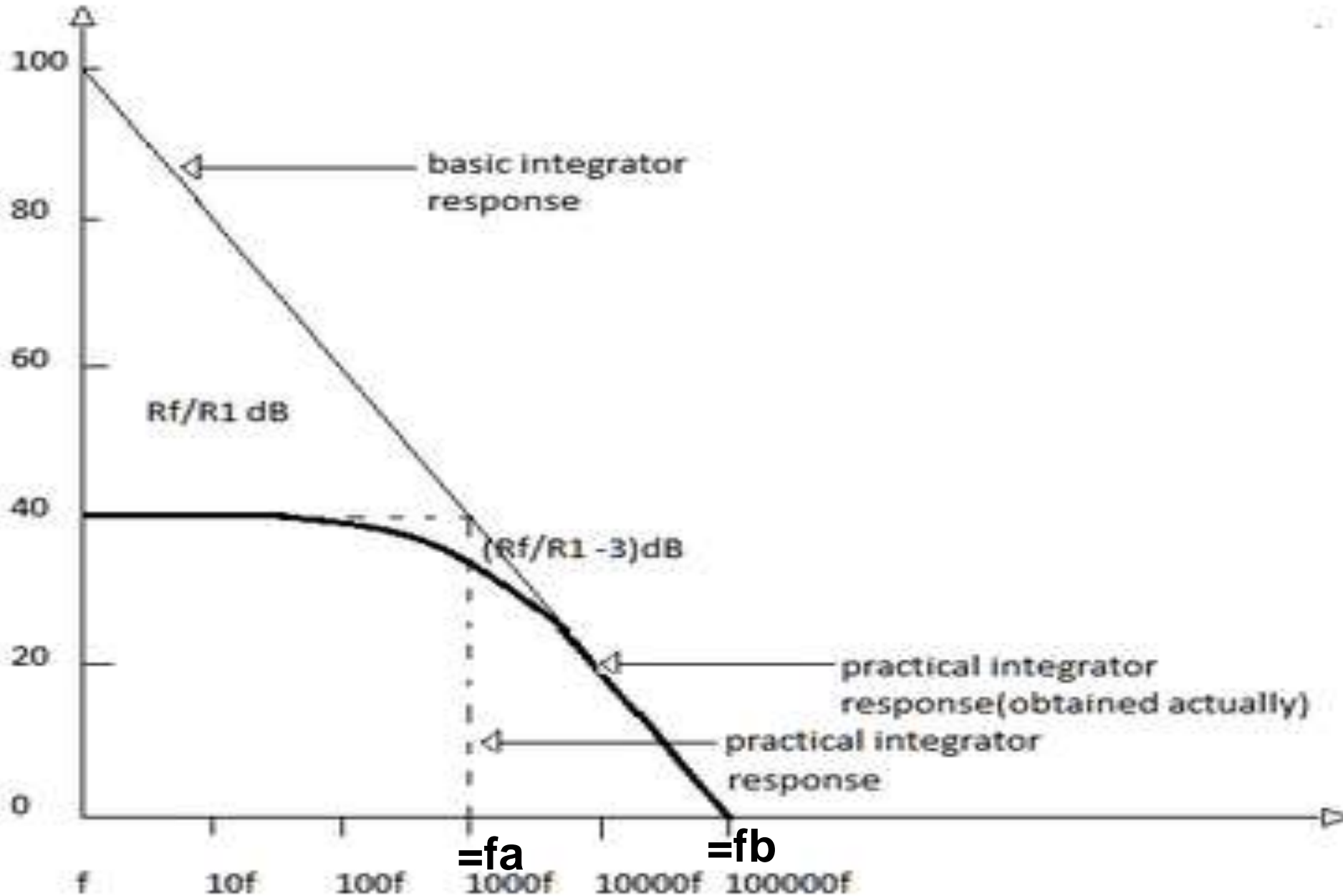


Practical Integrator



OP-AMP as a Practical Integrator

Frequency Response of Basic Integrator and Practical Integrator



For Ideal integrator
 $A_{vf} = V_o/V_i$
 $= X_c / R_f$
 $= 1/2\pi \cdot f \cdot C_1 \cdot R_f$
 as f \uparrow increases
 A_{vf} Decreases

Design steps of Practical Integrator:

Step1. In fig, f_b is the frequency at which the gain is 0 dB and is given by

$$f_b = 1 / 2\pi R_1 C_F$$

In this fig, f is some relative operating frequency and for frequencies f to f_a , gain R_f/R_1 remains constant.

However, after f_a the gain decreases at a rate of 20 dB/decade.

Step 2. The gain limiting frequency f_a is given by

$$f_a = 1 / 2\pi R_F C_F$$

Step3. generally the value of f_a and in turn $R_1 C_F$ and $R_F C_F$ should be selected such that $f_a < f_b$.

For eg. If $f_a = f_b/10$, then $R_f = 10 R_1$

Step 4. In fact the input signal will be integrated properly if the time period of the input signal $T \geq R_F C_F$.

Example 4.4

Consider the lossy integrator shown in Fig. 4.23 (c). For the component values, $R_1 = 10 \text{ k}\Omega$, $R_F = 100 \text{ k}\Omega$, $C_F = 10 \text{ nF}$, determine the lower frequency limit of integration and study the response for the inputs (i) sine wave, (ii) step input (iii) square wave.

Solution

For the given component values, the lower frequency limit of integration f_a is

$$f_a = \frac{1}{2\pi R_F C_F} = \frac{1}{2\pi \times 100 \text{ k}\Omega \times 10 \text{ nF}} = 159 \text{ Hz}$$

For 99% accuracy, the input frequency should be at least one decade above f_a i.e., 1.59 kHz. Accurate integration can be achieved beyond this frequency. However, there is an upper limit up to which circuit will integrate and it is determined by the frequency response of op-amp. However, as input frequency is increased, the output amplitude reduces as the gain of the integrator falls at a rate of 6 dB/octave.

(i) *Sine wave input:* For an input of 1 V peak sine wave at 5 kHz, the output v_o is

$$\begin{aligned} v_o(t) &= -\frac{1}{R_1 C_F} \int v_i(t) dt = -\frac{1}{10 \text{ k}\Omega \times 10 \text{ nF}} \int 1 \sin(2\pi \cdot 5000 t) dt \\ &= -10^4 \int \sin(2\pi \cdot 5000 t) dt = -\frac{10^4}{2\pi \cdot 5000} [-\cos(2\pi \cdot 5000 t)] \\ &= 0.318 \cos(2\pi \cdot 5000 t) \end{aligned}$$

(ii) **Step input:** If input is a step voltage $v_i = 1 \text{ V}$ for $0 \leq t \leq 0.3 \text{ ms}$, then the output voltage at $t = 0.3 \text{ ms}$ is

$$v_o = -\frac{1}{R_1 C_F} \int_0^{0.3 \text{ ms}} 1 \cdot dt = -\frac{1}{10 \text{ k}\Omega \times 10 \text{ nF}} \times t \Big|_{t=0}^{t=0.3 \text{ ms}}$$

$$= -10^4 \times 0.3 \times 10^{-3} = -3 \text{ V}$$

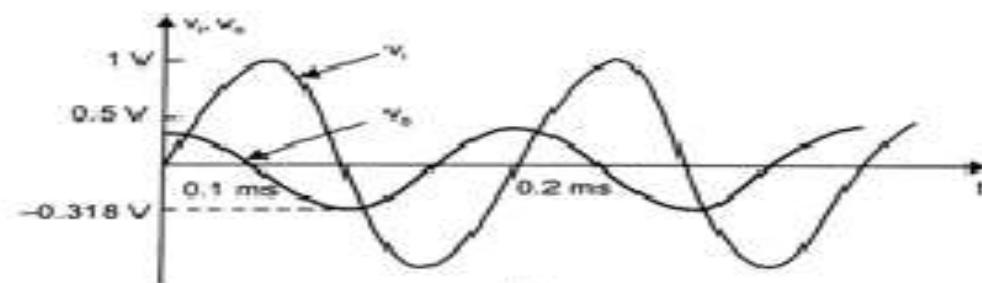
The output voltage is a ramp function with a slope of 10 V/ms and is shown in Fig. 4.25 (b).

(iii) **Square wave input:** The output waveform for an input of 5 KHz , 1 V peak square wave is shown in Fig. 4.25 (c).

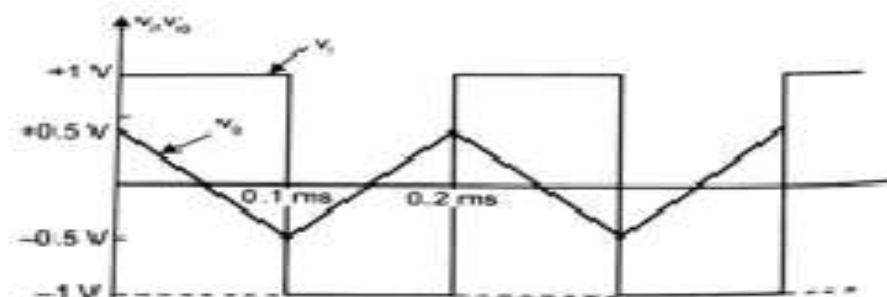
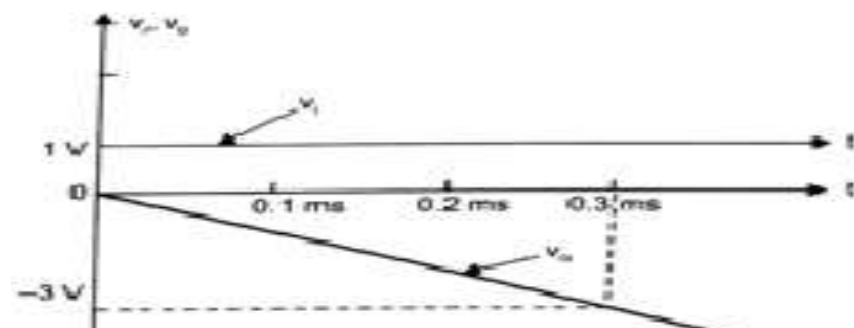
It can be seen that input is of constant amplitude of 1 V from 0 to 0.1 ms and -1 V from 0.1 ms to 0.2 ms . The output for each of these half periods will be ramps as seen above for step inputs. Thus, the expected output wave form will be a triangular wave. The peak value of the output for first half cycle is

$$v_o = -\frac{1}{R_1 C_F} \int_0^{0.1 \text{ ms}} 1 \cdot dt = -10^4 \times 0.1 \times 10^{-3} = -1 \text{ V}$$

This represents the total change in the output voltage over the first half cycle from 0 to 0.1 ms . Similarly, integration over the next half cycle produces a positive change of 1 V .

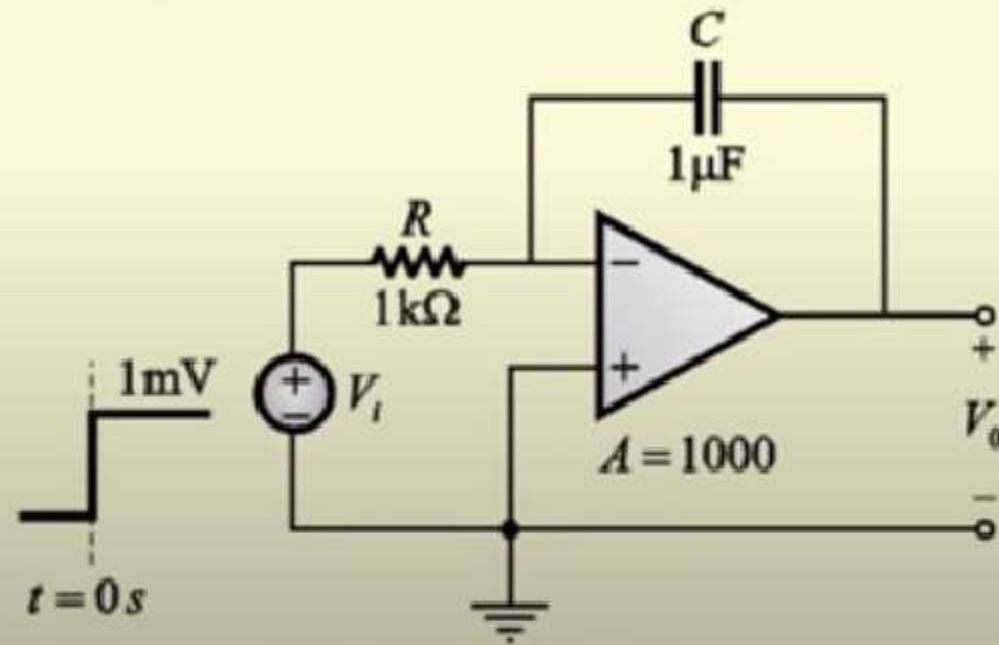


(a)



Question 6

The Op-Amp shown in the figure has a finite gain $A = 1000$ and an infinite input resistance. A step-voltage $V_i = 1\text{mV}$ is applied at the input at time $t = 0$ as shown in the figure. Assuming that the operational amplifier is not saturated, the time constant (in millisecond) of the output voltage V_o is [Set - 01]



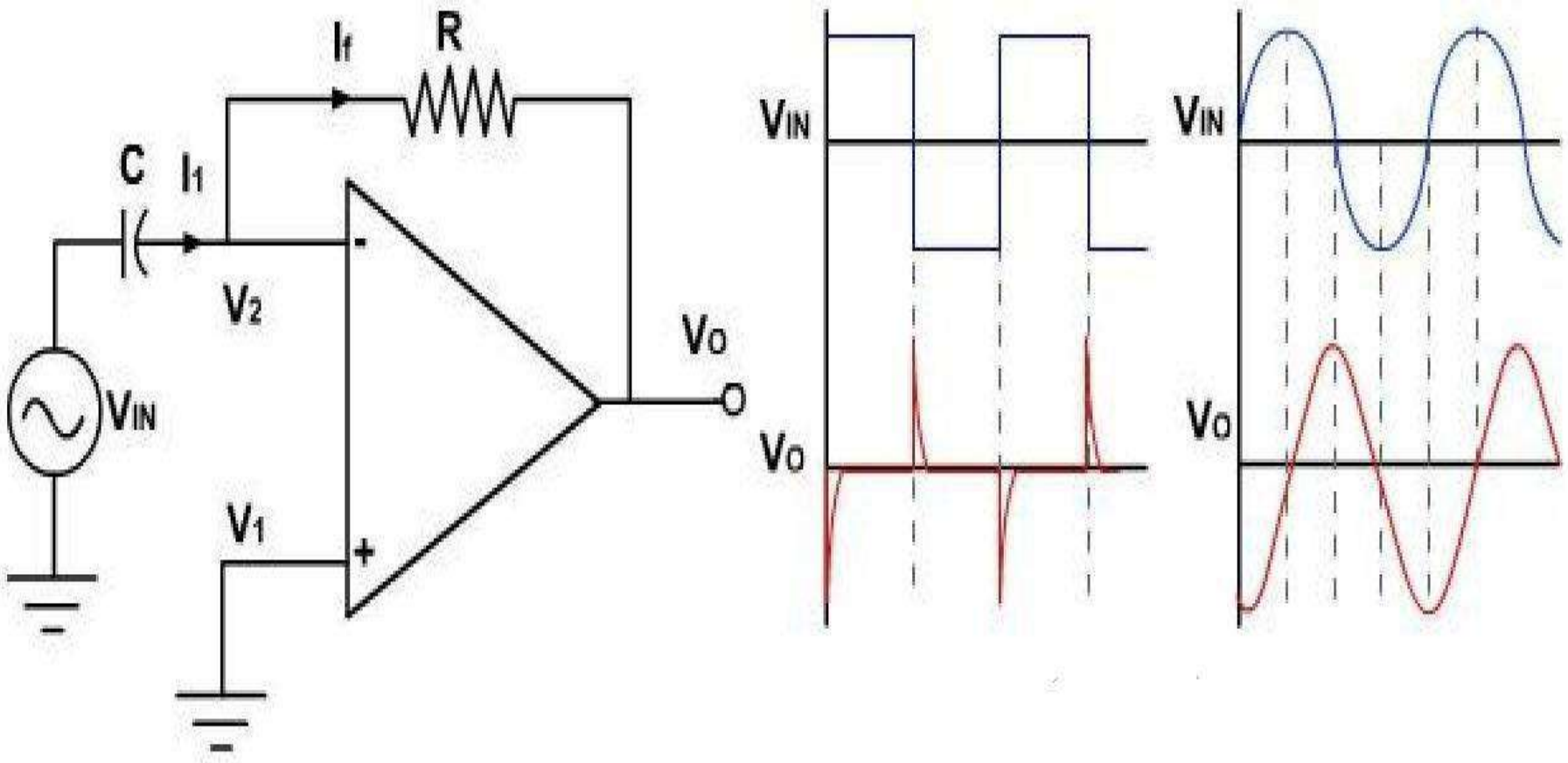
(A) 1001

(B) 101

(C) 11

(D) 1

Basic Differentiator



(b) Differentiator:

The expression for the output voltage can be obtained from KCL at node V2.

$$I_C = I_B + I_F \quad \text{but } I_B = 0,$$

So, $I_C = I_F$

$$C_1 \frac{d(V_{in} - V_1)}{dt} = (V_1 - V_o) / R_F$$

Since $V_1 = V_2 = 0$ ----- for Ideal OP-AMP

$$C_1 \frac{d(V_{in})}{dt} = -V_o / R_F$$

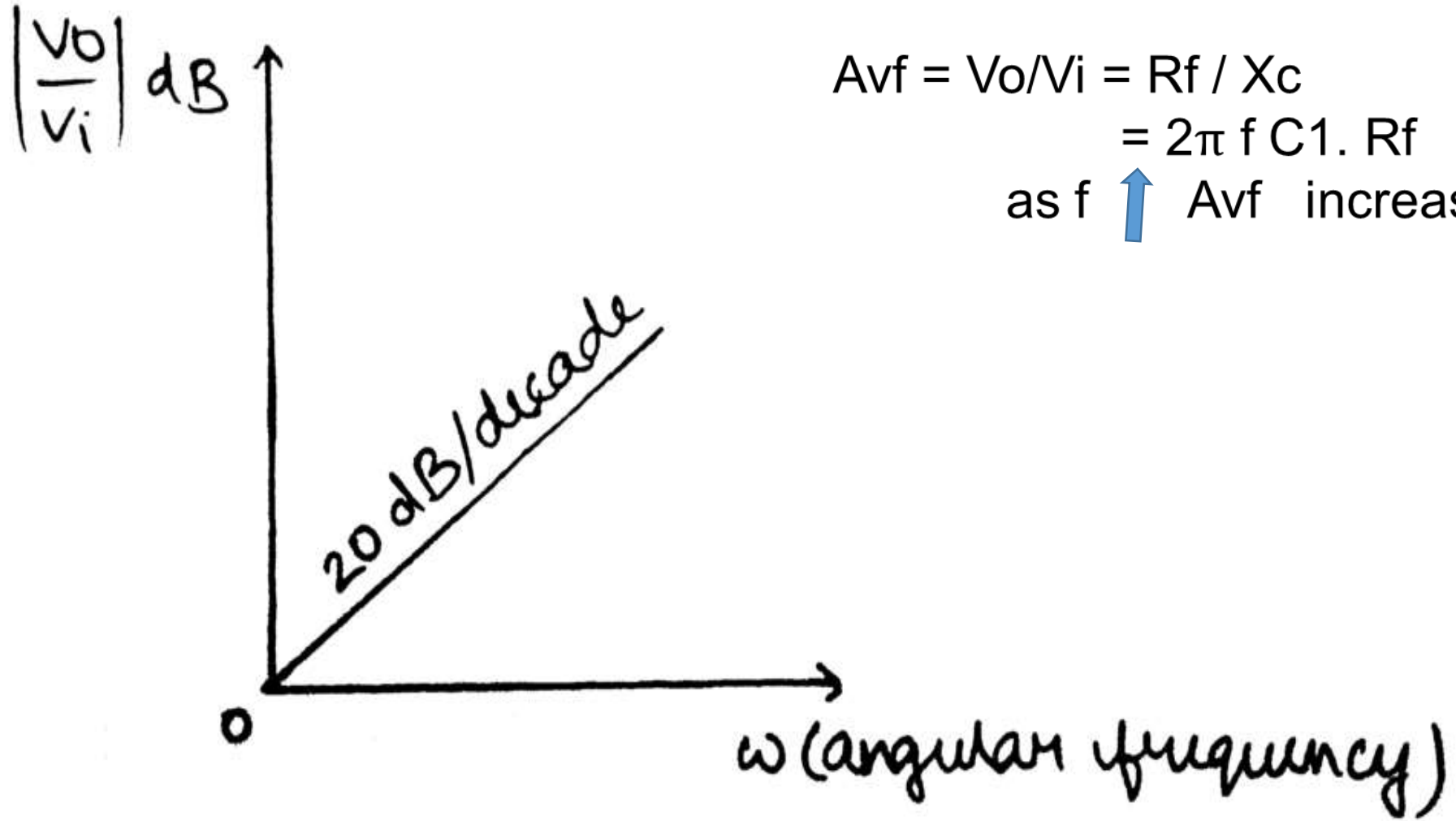
Rearranging the above equation, we get

So $V_o = -R_F * C_1 \frac{d}{dt}(V_{in})$ ----- this equation indicates that output

Voltage is purely the Derivative of Input signal if Time constant is

Chosen to be equal to 1. ie. Time constant = $\tau = R_F * C_1 = 1$ sec. with which
The Capacitor charges & discharges..

Frequency response of Basic Differentiator

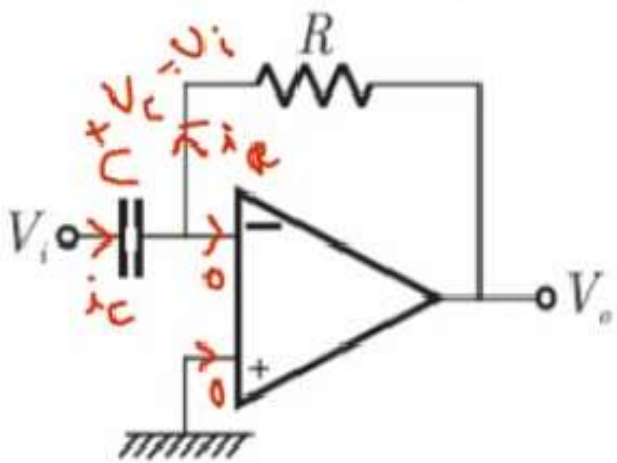


$$A_{vf} = V_o/V_i = R_f / X_c$$
$$= 2\pi f C_1 \cdot R_f$$

as $f \uparrow$ A_{vf} increases

Assume that the op-amp of the figure is ideal.

If v_i is a triangular wave, then v_o will be

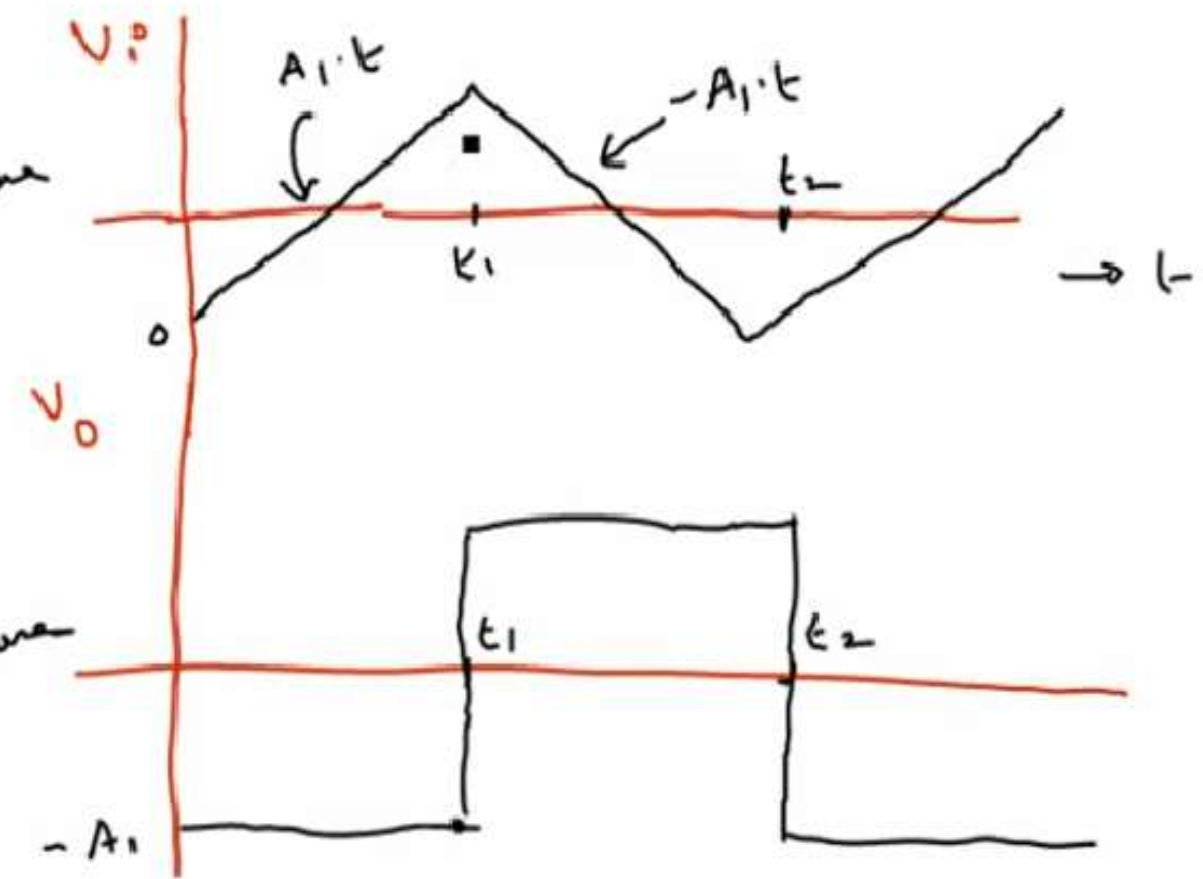


$V_o \propto -\frac{dV_i}{dt}$
Differentiator
triangular

$$i_c = i_R$$

$$C \cdot \frac{dv_c}{dt} = \frac{0 - V_o}{R}$$

$$C \cdot \frac{dv_i}{dt} = -\frac{V_o}{R} \Rightarrow V_o = -RC \frac{dV_i}{dt}$$



- (A) square wave
- (B) triangular wave
- (C) parabolic wave
- (D) sine wave

Advantages :

A small time constant is sufficient to cause differentiation of the input signal

Limitations :

At high frequencies: a) the simple differentiator circuit becomes unstable and starts to oscillate;

b) the circuit becomes sensitive to noise, that is, when amplified, noise dominates the input/message signal.

Practical differentiator :

In order to overcome the limitations of the ideal differentiator, an additional small-value capacitor C_F is connected in parallel with the feedback resistor R_F , which avoids the differentiator circuit to run into oscillations (ie, become unstable), and

a resistor R_1 is connected in series with the capacitor C_1 , which limits the increase in gain to a ratio of R_F/R_1 .

Since negative feedback is present through the resistor R_F , we can apply the virtual ground concept, ie. the voltage at the inverting terminal = voltage at the non-inverting terminal = 0.

V.IMP**

A resistance $R_{comp} (= R_1 \parallel R_F)$ is normally connected to the (+) input terminal to compensate for the input bias circuit.

A good differentiator may be designed as per the following steps:

1. Choose f_u equal to the highest frequency of the input signal. Assume a practical value of C_1 ($< 1 \mu\text{F}$) and then calculate R_F .
2. Choose $f_b = 10 f_u$ (say). Now calculate the values of R_1 and C_F so that $R_1 C_1 = R_F C_F$.

Example 4.3

- (a) Design an op-amp differentiator that will differentiate an input signal with $f_{max} = 100$ Hz.
- (b) Draw the output waveform for a sine wave of 1 V peak at 100 Hz applied to the differentiator.
- (c) Repeat part (b) for a square wave input.

Solution

(a) Select, $f_u = f_{max} = 100 \text{ Hz} = \frac{1}{2\pi R_F C_1}$

[from Eq. (4.71)]

Let

$$C_1 = 0.1 \mu\text{F}$$

then

$$R_F = \frac{1}{2\pi (10^2) (10^{-7})} = 15.9 \text{ k}\Omega$$

Now choose

$$f_b = 10 f_u = 1 \text{ kHz} = \frac{1}{2\pi R_1 C_1}$$

[from Eq. (4.74)]

Therefore,

$$R_1 = \frac{1}{2\pi (10^3) (10^{-7})} = 1.59 \text{ k}\Omega$$

Since

$$R_F C_F = R_1 C_1,$$

we get,

$$C_F = \frac{1.59 \times 10^3 \times 10^{-7}}{15.9 \times 10^3} = 0.01 \mu\text{F}$$

(b) $v_i = 1 \sin 2\pi(100)t$

From Eq. (4.69),

$$\begin{aligned} v_o &= -R_F C_1 \frac{dv_i}{dt} = -(15.9 \text{ k}\Omega) (0.1 \mu\text{F}) \frac{d}{dt} [(1 \text{ V}) \sin (2\pi) (10^2) t] \\ &= -(15.9 \text{ k}\Omega) (0.1 \mu\text{F}) (2\pi) (10^2) \cos [(2\pi) (10^2) t] = -0.999 \cos [2\pi (10^2) t] \\ &= -1 \cos [(2\pi) (10^2) t] \end{aligned}$$

The input and output waveforms are shown in Fig. 4.22 (a).

During the time periods for which input is constant at $\pm 1\text{ V}$, the differentiated output will be zero. However, when input transits between $\pm 1\text{ V}$ levels, the slope of the input is infinite for an ideal square wave. The output, therefore, gets clipped to about $\pm 13\text{ V}$ for a $\pm 15\text{ V}$ op-amp power supply as shown in Fig. 4.22 (b).

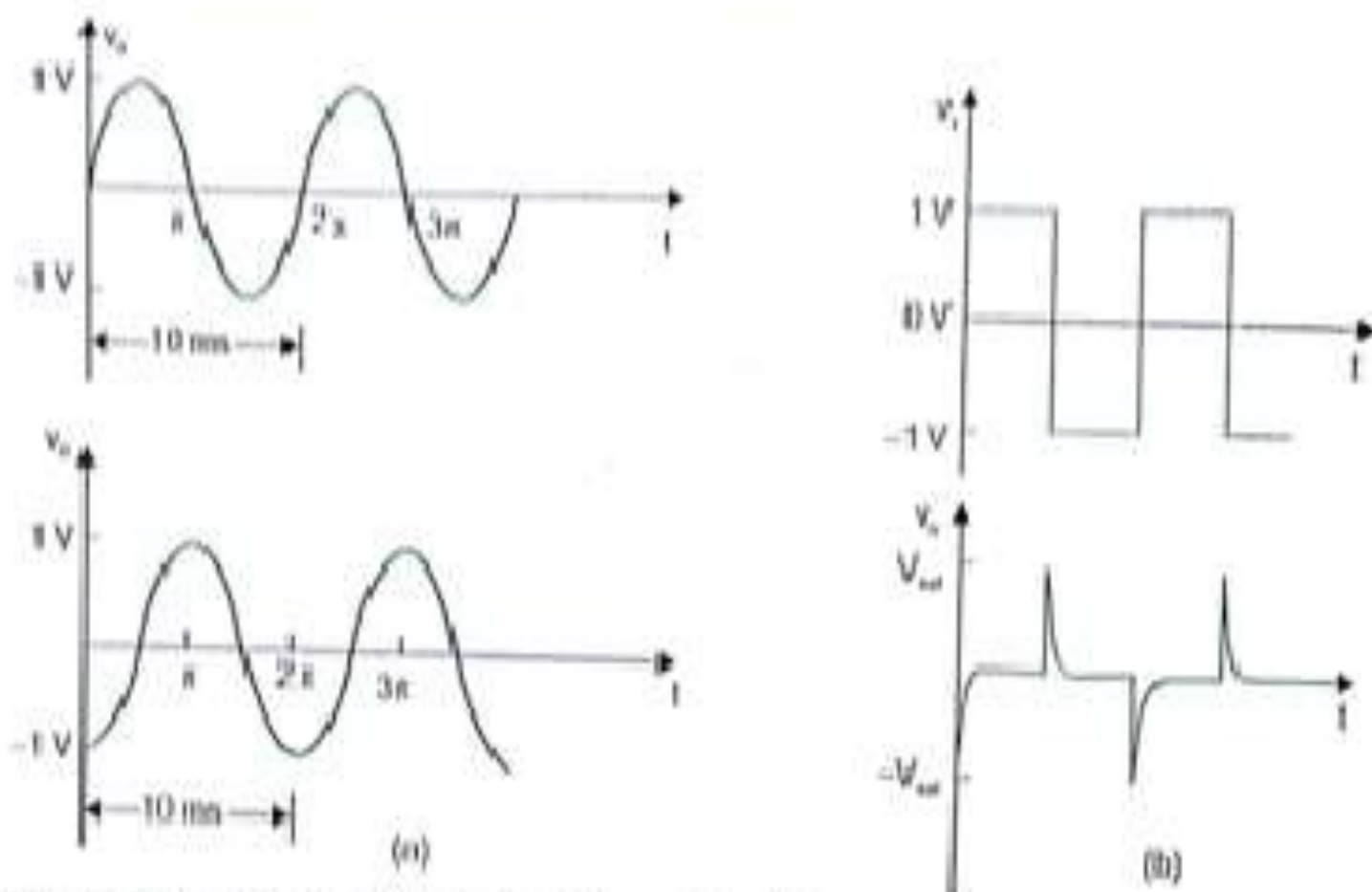
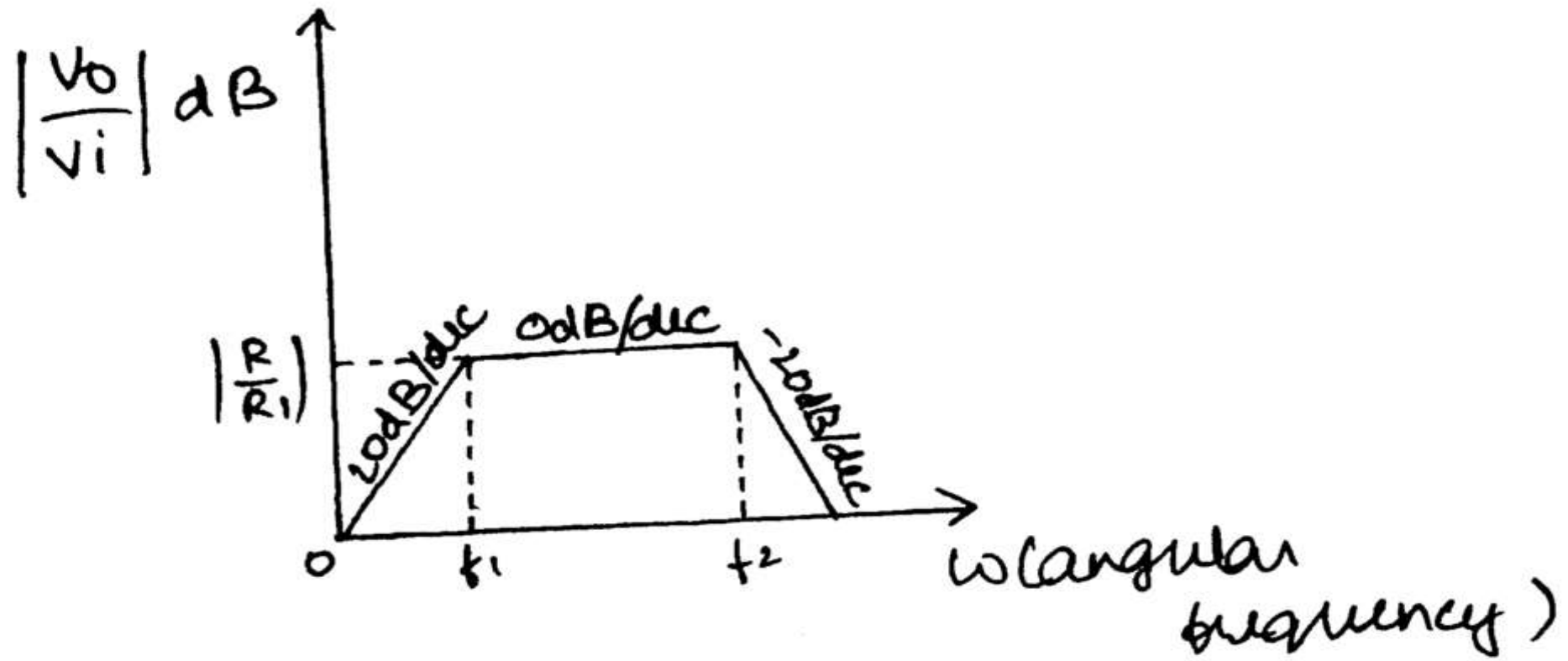
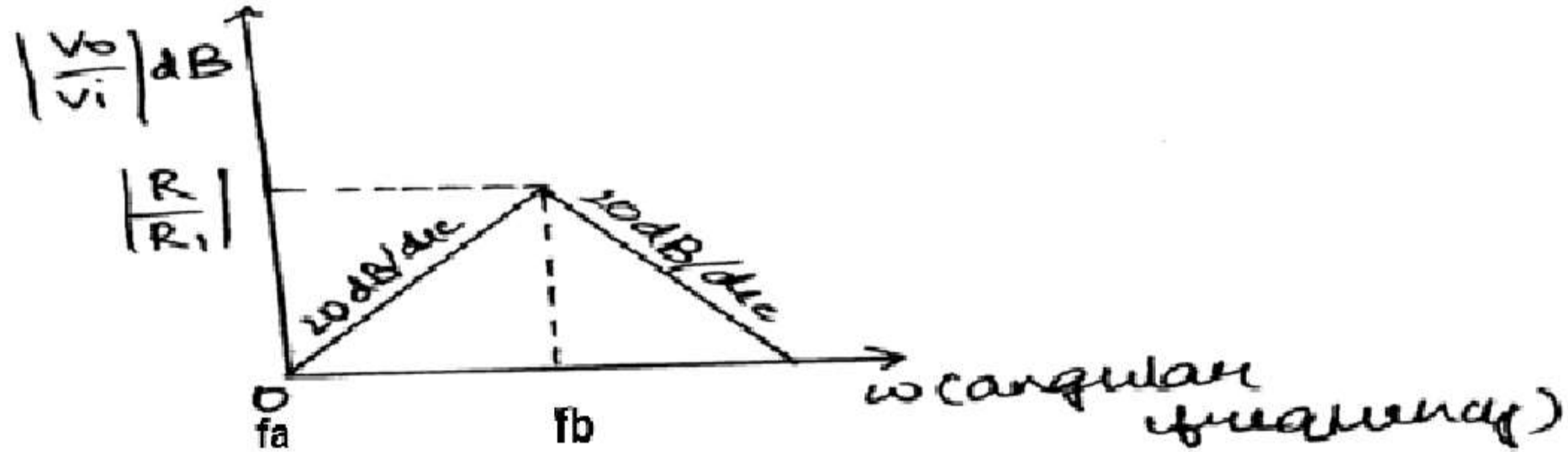


Fig. 4.22 (a) Sine-wave input and cosine output, (b) Square wave input and spike output



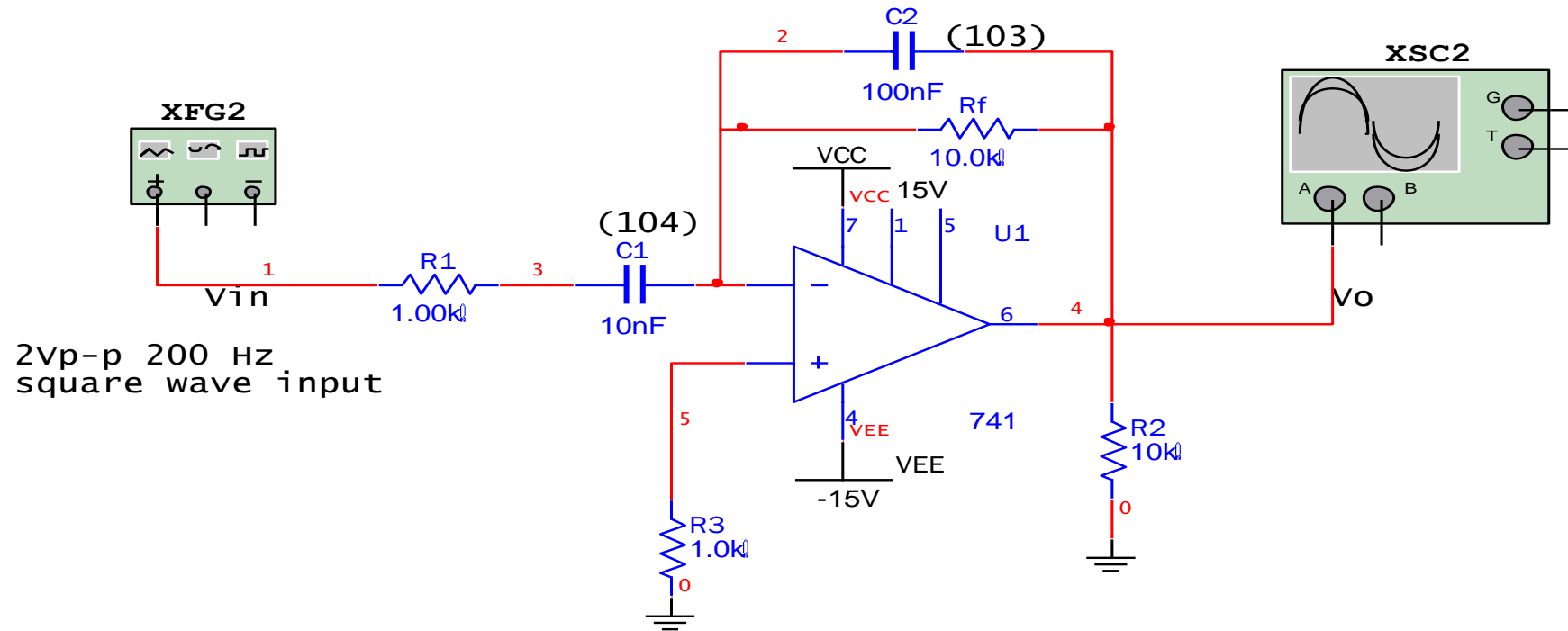
Frequency response of Practical Differentiator



From the above plot, we observe that:

- When $f < f_b$, the circuit acts as a differentiator;
- When $f > f_b$, the circuit acts as an [integrator](#).

Practical Differentiator



OP-AMP as a practical differentiator

Design steps of Practical Differentiator

Step1) **Select f_a equal to the highest frequency of the input signal to be differentiated at which the gain is 0 dB and is given by**

$$f_a = 1 / 2\pi R_F C_1 .$$

Step2) **Assume a value of $C_1 < 1 \mu\text{F}$, ie. $C_1 = 0.1 \mu\text{F} / 0.01\mu\text{F}$, calculate the value of R_F .**

Step3) **The gain limiting frequency f_b is given by**

$$f_b = 1 / 2\pi R_1 \cdot C_1$$

Step4) **Choose $f_b = 10 f_a$ and calculate the values of R_1 and C_f so that**

$$R_1 C_1 = R_F C_f$$

the input

A good differentiator may be designed as per the following steps:

1. Choose f_a equal to the highest frequency of the input signal. Assume a practical value of C_1 ($< 1 \mu\text{F}$) and then calculate R_F .
2. Choose $f_b = 10 f_a$ (say). Now calculate the values of R_1 and C_F so that $R_1 C_1 = R_F C_F$.

Example 4.3

- (a) Design an op-amp differentiator that will differentiate an input signal with $f_{\max} = 100$ Hz.
- (b) Draw the output waveform for a sine wave of 1 V peak at 100 Hz applied to the differentiator.
- (c) Repeat part (b) for a square wave input.

Solution

(a) Select, $f_a = f_{\max} = 100 \text{ Hz} = \frac{1}{2\pi R_F C_1}$ [from Eq. (4.71)]

Let $C_1 = 0.1 \mu\text{F}$,

then $R_F = \frac{1}{2\pi (10^2) (10^{-7})} = 15.9 \text{ k}\Omega$

Now choose $f_b = 10 f_a = 1 \text{ kHz} = \frac{1}{2\pi R_1 C_1}$ [from Eq. (4.74)]

Therefore, $R_1 = \frac{1}{2\pi (10^3) (10^{-7})} = 1.59 \text{ k}\Omega$

Since $R_F C_F = R_1 C_1$,

we get, $C_F = \frac{1.59 \times 10^3 \times 10^{-7}}{15.9 \times 10^3} = 0.01 \mu\text{F}$

(b) $v_i = 1 \sin 2\pi(100)t$
From Eq. (4.69),

$$\begin{aligned} v_o &= -R_F C_1 \frac{dv_i}{dt} = -(15.9 \text{ k}\Omega) (0.1 \mu\text{F}) \frac{d}{dt} [(1 \text{ V}) \sin (2\pi) (10^2) t] \\ &= -(15.9 \text{ k}\Omega) (0.1 \mu\text{F}) (2\pi) (10^2) \cos [(2\pi) (10^2) t] = -0.999 \cos [2\pi (10^2) t] \\ &= -1 \cos [(2\pi) (10^2) t] \end{aligned}$$

The input and output waveforms are shown in Fig. 4.22 (a).

the input

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2. Choose $f_b = 10 f_a$ (say). Now calculate the values of R_1 and C_F so that $R_1 C_1 = R_F C_F$.

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Solution

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Since $R_F C_F = R_1 C_1$,

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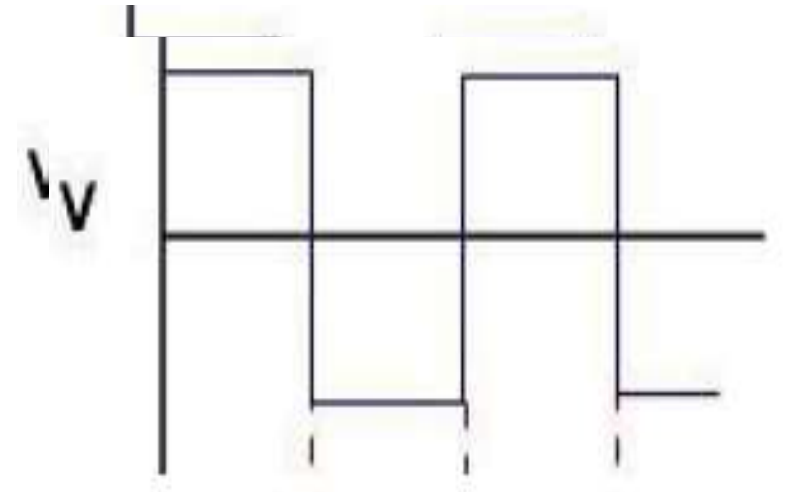
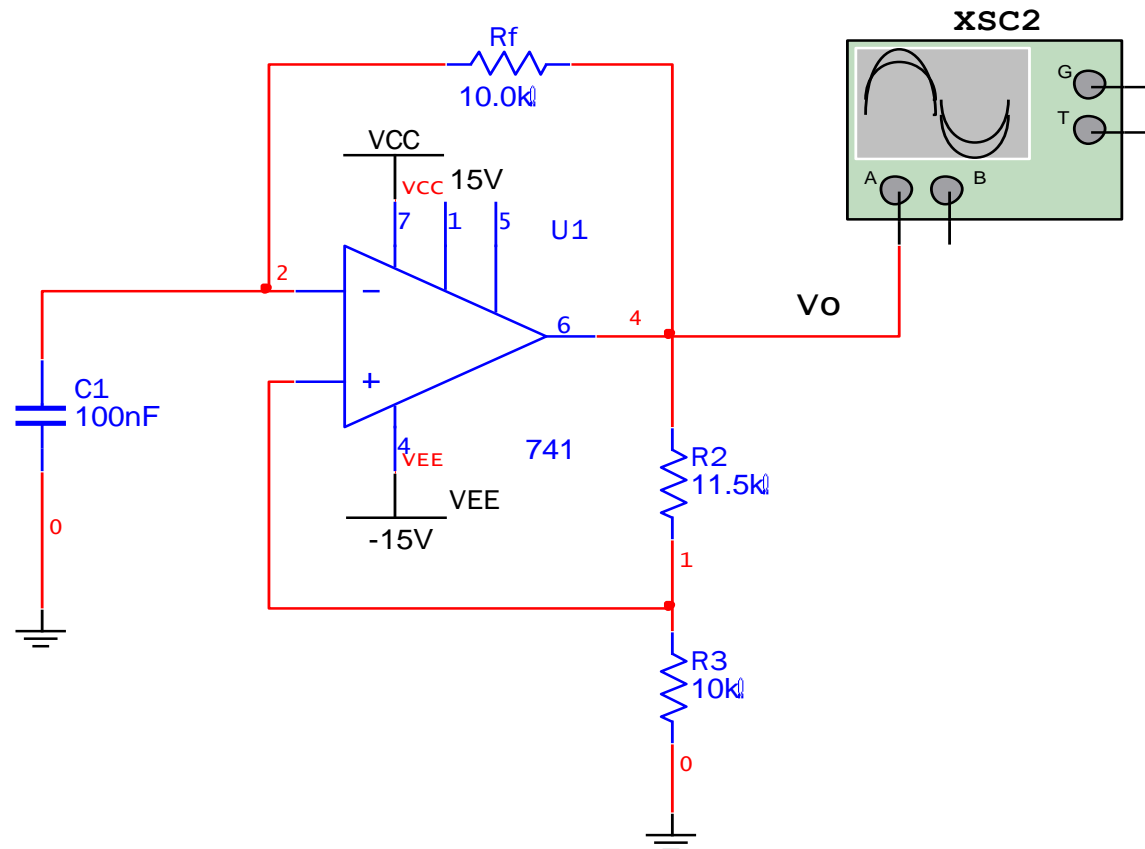
The input and output waveforms are shown in Fig. 4.22 (a).

Applications:

- 1) The differentiator circuit is essentially a [high-pass filter](#).
- 2) It can generate a [square wave](#) from a [triangle wave](#) input and
- 3) Produce alternating-direction voltage spikes when a square wave is applied. In ideal cases, a differentiator reverses the effects of an [integrator](#) on a waveform, and conversely.
- 4) Hence, they are most commonly used in [wave-shaping circuits](#) to detect high- frequency components in an input signal. Differentiators are an important part of electronic [analogue computers](#) and analogue [PID controllers](#).
- 5) They are also used in [frequency modulators](#) as rate-of-change detectors.

A passive differentiator circuit is one of the basic [electronic circuits](#), being widely used in circuit analysis based on the [equivalent circuit](#) method.

To design a square wave generator using OP-AMP 741 IC for $f=500\text{Hz}$.



OP-AMP as a square wave generator

For generation of square wave the OPAMP is forced to operate in its saturation region. The voltage available at non-inverting input of OPAMP is obtained by potential-divider action.

$$V_1 = (R_2/R_1 + R_2) V_o$$

So, $V_1 = \beta * V_o$, where $\beta = R_2/(R_1 + R_2)$

The o/p V_o is also feedback to inverting I/p terminal where I/p voltage obtained is given by

$$V_c = (1/RC) \int V_o dt.$$

Whenever $V_c > V_{ref} > \beta V_o$ Switching takes place and square wave output is available from comparator developed by op-amp.

Theoretically: $f_o = 1/2RC$

Practically: $f_o = 1/T$

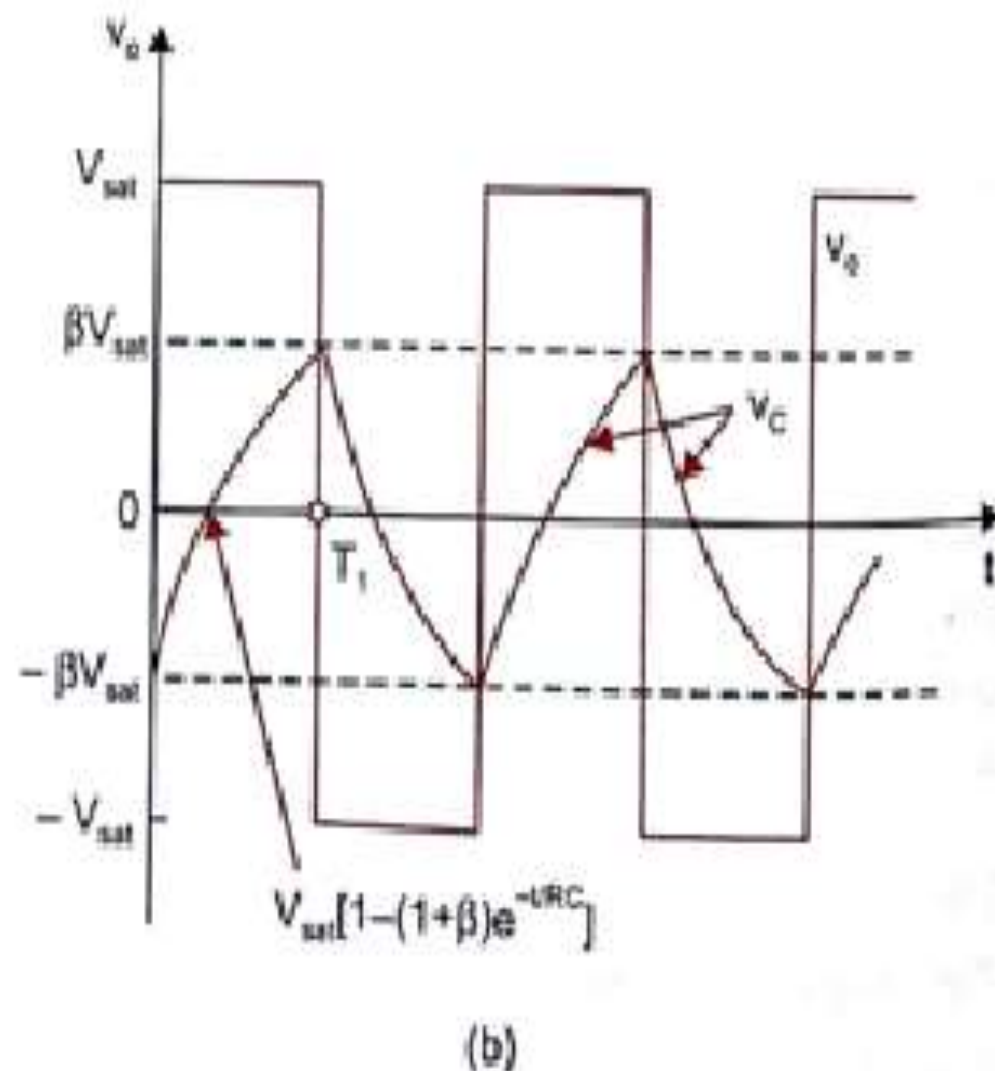
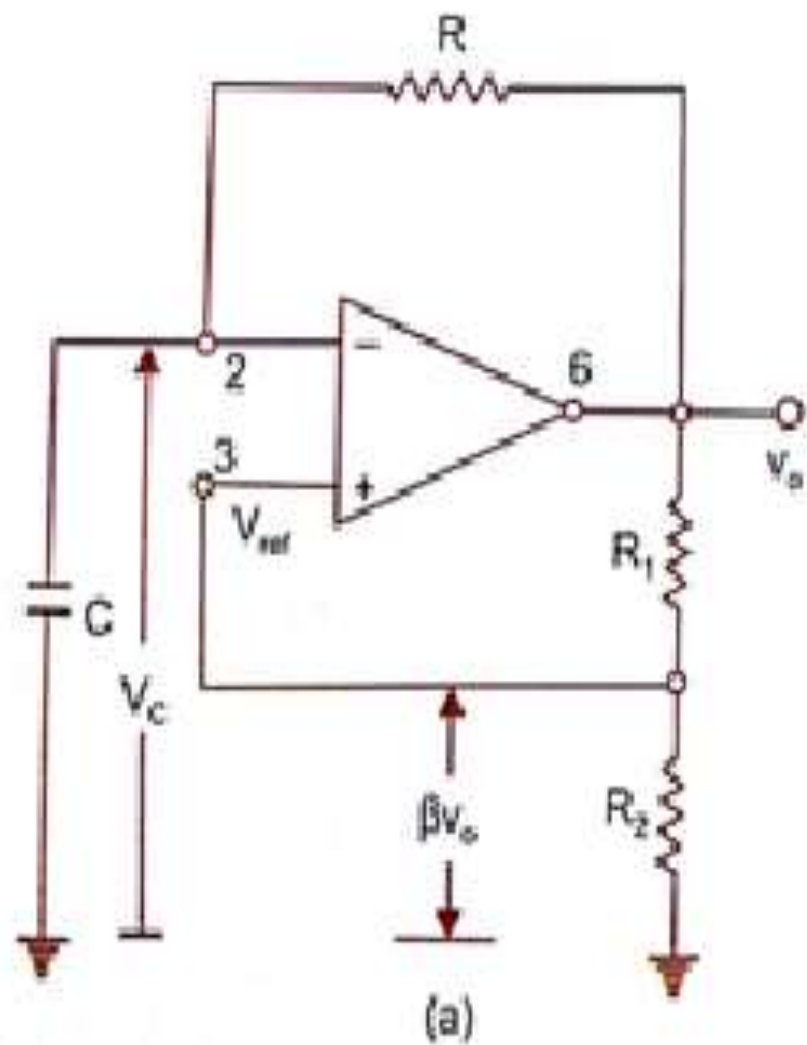


Fig. 5.10 (a) Simple op-amp square wave generator (b) Waveforms

The frequency is determined by the time it takes the capacitor to charge from $-\beta V_{\text{sat}}$ to $+\beta V_{\text{sat}}$ and vice versa. The voltage across the capacitor as a function of time is given by,

$$v_c(t) = V_f + (V_i - V_f)e^{-t/RC} \quad (5.4)$$

where, the final value, $V_f = +V_{\text{sat}}$
and the initial value, $V_i = -\beta V_{\text{sat}}$

Therefore,

$$v_c(t) = V_{\text{sat}} + (-\beta V_{\text{sat}} - V_{\text{sat}})e^{-t/RC} \quad (5.5)$$

or

$$v_c(t) = V_{\text{sat}} - V_{\text{sat}}(1 + \beta)e^{-t/RC} \quad (5.5)$$

At $t = T_1$, voltage across the capacitor reaches βV_{sat} and switching takes place. Therefore,

$$v_c(T_1) = \beta V_{\text{sat}} = V_{\text{sat}} - V_{\text{sat}}(1 + \beta)e^{-T_1/RC} \quad (5.6)$$

After algebraic manipulation, we get,

$$T_1 = RC \ln \frac{1 + \beta}{1 - \beta} \quad (5.7)$$

This give only one half of the period.

Total time period

$$T = 2T_1 = 2RC \ln \frac{1 + \beta}{1 - \beta} \quad (5.8)$$

and the output wave form is symmetrical.

If $R_1 = R_2$, then $\beta = 0.5$, and $T = 2RC \ln 3$. And for $R_1 = 1.16R_2$, it can be seen that

$$T = 2RC$$

or

$$f_0 = \frac{1}{2RC}$$

The output swings from $+V_{\text{sat}}$ to $-V_{\text{sat}}$, so,

$$v_o \text{ peak-to-peak} = 2V_{\text{sat}} \quad (5.9)$$